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Joint optimization of production and maintenance strategies considering a dynamic sampling strategy for a deteriorating system

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ABSTRACT

This paper presents a control policy of integrated production, maintenance and quality control planning for a continuous production system subject to quality deterioration. The system under study is composed of an unreliable machine that produces one part type satisfying customers demand. The machine can fail at any time and is subject to quality deterioration, and so a preventive maintenance and quality control policies are proposed to decrease the rate of defectives and to increase the system availability. The proposed production policy incorporates production thresholds, which regulate the machine production rate. Traditional sampling inspections standards such as ANSI/ASQC Z1.4 and ISO 2859 have addressed a dynamic quality control level to face quality deterioration. However, these standards are based only on quality considerations, disregarding the economic aspects and the interactions with production and maintenance management in the design of sampling plans. Thus, the proposed integrated model analyses in detail the effect of such dynamic sampling strategy and relevant interactions with production and maintenance strategies. The main objective of the paper is to determine an appropriate production policy as well as the preventive maintenance and the quality control rates in order to minimize the expected average incurred cost and satisfy at the same time a quality constraint. Given the high flexibility and capacity to model complex manufacturing systems, a combination of simulation modeling and optimization techniques are used to determine a solution for this stochastic and constrained problem. In addition, numerical examples and an extensive sensitivity analysis are conducted to illustrate the proposed control approach. Furthermore, we compared the proposed integrated policy with three common policies from the literature. Such study serve us to highlight the effectiveness of the approach, since the proposed integrated policy led to considerable cost savings.

1. Introduction

Quality, production planning and maintenance plays a critical role in modern production systems and it is clear that their mutual interactions should not be neglected while managing production systems. Unfortunately, only a few contributions address problems under an integrated view. According to Colledani et al. (2014), limited literature exists in the development of new models that allow companies to identify strategic targets in these three key functions and balance them towards a desired equilibrium. Thus, the research conducted in this paper aims to propose an integrated model which avoids sub-performing unbalance solutions that privileges one or two of the functions while decreasing the overall production system performance.

A common feature observed from the manufacturing system domain is that it focuses on static sampling plans whose parameters do not change over time. However, we state that in many complex manufacturing processes, such as in electronics, automobile and chemical industries, deterioration is a common phenomenon, which certainly have a significant impact on the control policy. Thus, we conjecture in this paper that the quality control policy, must be adjusted continuously in function of the level of deterioration of the production system.

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Based on these observations, we can observe that much research is needed, since the industrial sector demands advanced engineering scheduling methods for joint production, quality control and maintenance planning incorporating in the model the deterioration phenomenon with the aim to keep companies profitable.

The rest of the paper is organized as follows. The literature review is presented in Section 2. The industrial context is presented in Section 3. The notation, assumptions and description of the production system under study is detailed in Section 4. The model formulation, the joint control policy and the optimization problem under study are presented in Section 5. Afterwards, an approach based on simulation-optimization is detailed in Section 6. Moreover, in Section 7 a simulation model and its validation are presented. A numerical example is analyzed in Section 8 to illustrate the proposed approach. In Section 9 we performed an extensive sensitivity analysis. Further, in Section 10 we present a comparative study where we highlighted the efficiency of our policy compared with alternative control policies common on the literature. Section 11 concludes de paper.

2. Literature review

To adequately situate our proposed integrated model, we present an overview of the literature from five relevant research topics, which represent the main fields that have been addressed in the last years in the production system domain. Particularly, we consider models that have focused in: (i) the production and quality relationship, (ii) quality information feedback and maintenance strategies, (iii) quality control inspection strategies, (iv) studies on the joint quality, production and maintenance planning, and (v) deterioration models. In the next paragraphs we discuss these five topics.

(i) The relationship between production and quality aspects was explored by Inman, Blumenfeld, Huang, and Li (2003), where they suggested several important issues about this interaction, also they highlighted the fact that increasing quality is mandatory for modern companies. We find in the literature analytical models such as in the paper of Kim and Gershwin (2005, 2008), where they presented a method for the performance analysis of production systems. In particular, they analyzed how production system design, quality and productivity are inter-related in such systems. In the same line Colledani and Tolio (2011) presented an analytical method for evaluating the performance of production systems in which the behavior of the machine is monitored by statistical control charts. Their method considered the presence of inspection and integrated stations in the line that produces defectives. The mutual inter-relation of quality and production measures, denotes a growing area that has spurred significant research as the paper of Mhada, Hajji, Malhame, Gharbi, and Pellerin (2011). These authors presented analytical expressions for the optimal production threshold and the optimal cost of a production control problem of a failure-prone manufacturing system that produces a random fraction of defective items. Nourelfath, Nahas, and Ben-Daya (2016) addressed the joint selection of the optimal values of production plan and the maintenance policy while taking into account quality related costs. As can be noted from the presented papers, the area of research that jointly addresses quality-production is very active. Currently, the study of such relationship, has devised new directions towards the consideration of maintenance strategies, since the fundamental functions of production-quality and maintenance are strongly interrelated. Unfortunately, the amount of literature on this field is limited.

(ii) Several production paradigms have been proposed to analyze the inter-relation between quality information feedback and maintenance strategies. Given that quality information feedback could provide useful information for process monitoring and maintenance-decision making and it may lead to significant cost savings. Preventive maintenance based on quality information feedback has fostered considerable research in recent years since it represents an alternative solution to palliate the system degradation. In this context, Njike, Pellerin, and Kenné (2011) used an iterative feedback based on the quantity of defective products to determine the optimal maintenance and production planning, since they proposed that defective products are a consequence of global manufacturing system deterioration. In the paper of Lu, Zhou, and Li (2016), it was integrated quality improvement into preventive maintenance decision-making, also an integrated reliability model is built for the machine based on the proportional hazard model taking into account the effects of the degradation states of quality-related components. Furthermore, Shrivastava, Kulkarni, and Vrat (2016) considered the joint optimization of preventive maintenance and quality control policy, their model enables the determination of the optimal value of the parameters of a control chart and the preventive maintenance interval. From these models we note the relevance of quality information for process condition monitoring and maintenance decision-making.

(iii) Additionally, a limiting assumption observed in the literature is the consideration that quality control inspection has a negligible duration and cost as in the papers of Radhoui, Rezg, and Chelbi (2010). In this study the authors determined a preventive maintenance strategy based on the rate of non-conforming units detected on an automated quality control that disregards the inspection duration and cost. On another hand, in the paper of Mhada, Malhamé, and Pellerin (2013), it was addressed the problem of the joint determination of buffer sizing and inspection station placement for an unreliable production system which includes two inspection stations, one station dedicated to the inspection of finished parts, while the location of the other station is chosen so to optimize the total average cost comprising the storage cost, possible shortage and the inspection cost; however the inspection duration is not considered in their study. Sahnoun, Bettayeb, Bassetto, and Tollenaere (2014) defined an optimized quality control plan that reduced required capacity of control while maintaining enough trust in quality control. They observed that significant savings could have been obtained by using an optimized sampling plan. Nevertheless, they disregarded the impact of inspection duration in their analysis. Additionally several sampling strategies exist and can be classified according to their capability to integrate the factory variability, where the focus has changed from static sampling to dynamic sampling such as the paper of Lee (2002). Who presented a dynamic sampling strategy for a semiconductor industry that dynamically determined the sampling locations and sampling size on the basis of defect pattern change for faster detection of any abnormality. Rodriguez-Verjan, Dauzére- Pérès, and Pinaton (2015) proposed other dynamic sampling strategy, where the difference with other techniques is that in their model lots are selected in real time, according to the information that can be obtained by inspecting lots and based on the current production state. Their model also calculates the inspection capacity required in order to satisfy a given quality limits. From these papers, it is evident that dynamic sampling effectively utilizes the inspection capacity for quicker excursion detection and increases the throughput of inspection without affecting the quality of inspection. However, the interaction of dynamic sampling strategies with production and preventive maintenance planning has not been studied.

(iv) The increasing emphasis on sustainable production requires high system availability, excellent product quality and productivity along the production system life cycle. Several contributions on this domain have focused on the relationship between production and maintenance functions as in the paper of Kenné and Gharbi (2004) and Ramirez-Restrepo, Hennequin, and Aguezzoul (2016) where they proposed a method to determine optimal maintenance and production rates for an unreliable manufacturing system. In the paper of Dahane, Clementz, and Rezg (2010) it was considered the problem of the joint maintenance management and production control for a production unit that satisfies a constant demand and at the same time can be allocated to perform additional productions tasks to a contractor production system. In their model they determined the impact of an unforeseen extension of these additional production tasks on the generated cost. Despite the relevance of these papers, it should be point out that there is a very limited number of integrated models in the literature that address mutual relations among quality, production and maintenance planning. For example, Colledani and Tolio (2012) analyzed the production rate of conforming parts in manufacturing systems with reduction of their product quality and preventive maintenance. Their model determined the state threshold that activates preventive maintenance, whenever the machine is detected to be in an undesired state characterized by degrading performance. Further, Rivera-Gómez, Gharbi, and Kenné (2013) investigated the joint production and maintenance planning for an unreliable machine subject to quality deterioration. They determined the optimal production threshold and the optimal overhaul strategy that mitigate the presence of defectives. Recently, Bouslah, Gharbi, and Pellerin (2016) considered the problem of integrating the batch production strategy and quality control that is performed using a single acceptance-sampling plan by attributes. Also they considered that maintenance is undertaken once the proportion of defectives in a rejected lot reaches a given threshold. Other integrated model was presented by Bouslah, Gharbi, and Pellerin (2018), who studied a multistage manufacturing system consisting of two unreliable machines where defectives products manufactured in upstream processes have a significant impact on the production system reliability. Also they proposed a control policy that defines the production thresholds, the level of quality control implemented in the machines and the critical age to conduct preventive maintenance. Lopes (2018) studied the influence of a quality inspection policy for an imperfect manufacturing system with defective production. They considered that the inspection policy is imperfect and that defective items detected on inspection are sent for reworking and preventive maintenance is performed after each production cycle. As can be noted, more research is needed in this domain to fully integrate production-quality and maintenance functions in a joint control strategy.

(v) In the literature, several attempts have been made to incorporate deterioration in the optimal strategies. For instance, Chouikhi, Khatab, and Rezg (2014) addressed a condition-based maintenance strategy for a system subject to deterioration, which impacts the product quality. To control this deterioration, inspections are performed and after which the system is preventively replaced. In Kouedeu, Kenné, Dejax, Songmene, and Polotski (2015) it was studied the impact of imperfect repairs for the joint analysis of the optimal production and maintenance planning of a deteriorating manufacturing system. In their model corrective and preventive maintenance are determined based on the level of deterioration of the production unit. He, Gu, Chen, and Han (2017) considered a manufacturing system whose equipment is in a state of continuous degradation during operation. Their paper proposed a predictive maintenance strategy, where key process variables are identified and integrated into the evaluation of the equipment state. Kang and Subramaniam (2018) integrated control of maintenance and production in a deteriorating manufacturing system. Their model uses the downtime of machines as potential opportunities to perform maintenance on other machines. Another model targeting product reliability degradation was proposed by He et al. (2019), who applied preventive maintenance and a time-between-events (TBE) chart to detect any undesired machine state deteriorations. Their model determines the critical state, PM interval and lower control limit of TBE chart. It is evident from these papers that the influence of the deterioration process on the joint design of production, quality and maintenance strategies has been commonly disregarded. Nevertheless, the influence of the deterioration process on the control policy certainly may be significant.

Summing up, we present in Table 1 a comparison of the literature with the contribution of this paper. The lines I-V of Table 1 shows thematically the papers that have been discussed in this section, while their columns present a set of key features that highlight in such papers.

From the articles presented in the previous paragraphs, it is clear that few papers have considered the effects of a deterioration process on the three key functions of inventory management, maintenance and quality sampling inspection. Thus, in this paper a joint integrated model is proposed, in contrast to previous research where quality decisions are separated from production and maintenance. In particular, our model aims to extend existing literature, mainly the papers of Rivera-Gómez et al. (2013) and Bouslah et al. (2016, 2018). With the difference that we focus in the following issues: (i) determination of a quality strategy, which takes into account the economic aspect and the influence on production, inventory and maintenance management. (ii) Implementation of a quality sampling plan that continuously evolves in function of the level of deterioration of the machine. (iii) Consideration of non-negligible duration and cost of inspection and rectification activities. (iv) Consideration of a quality level constraint in the joint determination of the production, maintenance and quality control strategies. (v) Consideration of the impact of a deterioration process on the production, quality and maintenance control parameters. The combination of these set of characteristics has not been simultaneously considered in the literature before. Further the study of these issues are a need of industrial manufacturers since the functions of production, quality and maintenance are key for economic success of organizations.

3. Industrial context

Our model can be applied in production systems, where machines are subjected to random failures, their production rates can be controlled and the system evolves with stochastic dynamics deriving in deterioration. Such process challenges performance operation, as in machining centers, grinders, and other machining tools. Such systems normally have a large number of components that deteriorates over time; thus causing the machine to experience negative effects of its performance, (Dehayem-Nodem, Kenné, & Gharbi, 2011a). However, the impact of quality issues must considered in such deterioration process. Additionally, we realize that if production is carried on with such deteriorating systems, it may accelerate the machine degradation, and limit its production capacity. Once the production system is unable to satisfy customer demand, the increase in additional costs due to deterioration force companies to devise effective and efficient countermeasures to face the effects of such deterioration process. Several manufacturing sectors such as in the electronics, automobile and chemical industries experience deterioration phenomenon, (Kouedeu et al., 2015). Unfortunately, the field of production-quality-maintenance has disregarded the effects of such deterioration process on the control strategy (Colledani et al., 2014). The proposed integrated model presented in this paper is suitable for unreliable production systems subject to deterioration and determines simultaneously production, quality control and maintenance policies. Below, we proposed an integrated model and develop appropriate techniques for its solution.

4. Notations, assumptions and problem statement

In this section we introduce the notations used in the formulation of the developed model; also we present the assumptions of the model and the problem statement.

Table 1
Summary of literature review.

	Performance indices analysis	Deteriorating system	Production control	Preventive maintenance	Quality control policy	Quality level constraint	Non-negligible inspection duration	Static sampling strategy	Dynamic sampling strategy	Stochastic optimal control	Repairs-based indice
I. Production-quality relationship											
Kim and Gershwin (2008)	✓	✓									
Kim and Gershwin (2005)	✓	✓									
Colledani and Tolio (2011)	✓	✓			✓						
Mhada et al. (2011)			✓							✓	
Nourelfath et al. (2016)			✓	✓	✓						
II. Quality information feedback and maintenance strategies											
Njike et al. (2011)		✓	✓	✓						✓	✓
Lu et al. (2016)		✓		✓							
Shrivastava et al. (2016)		✓		✓	✓						
III. Quality control inspection											
Radhoui et al. (2010)		✓	✓	✓	✓						
Mhada et al. (2013)			✓		✓					✓	
Sahnoun et al. (2014)		✓			✓			✓			
Lee (2002)					✓				✓		
Rodriguez-Verjan et al. (2015)			✓		✓	✓			✓		
IV. Quality, production and maintenance planning											
Kenné and Gharbi (2004)			✓	✓						✓	
Ramirez-Restrepo et al. (2016)			✓	✓							
Dahane et al. (2010)			✓	✓							
Colledani and Tolio (2012)	✓	✓		✓	✓		✓				
Rivera-Gómez et al. (2013)		✓	✓	✓						✓	✓
Bouslah et al. (2016)		✓	✓	✓	✓	✓	✓	✓		✓	
Bouslah et al. (2018)		✓	✓	✓	✓	✓	✓	✓		✓	
Lopes (2018)			✓	✓	✓	✓	✓	✓		✓	
V. Deteriorating systems											
Chouikhi et al. (2014)		✓		✓							
Kouedeu et al. (2015)		✓	✓	✓						✓	✓
He et al. (2017)		✓		✓	✓						
Kang and Subramaniam (2018)		✓	✓	✓						✓	
He et al. (2019)		✓		✓	✓						
The proposed model		✓	✓	✓	✓	✓	✓		✓	✓	✓

4.1. Notations

The model under consideration is based on the following notations:

$x(t)$	Inventory level at time t
$n(t)$	Current number of failures at time t
d	Market demand rate of products
$\alpha(t)$	State of the machine at time t
τ_p	Unit production duration
τ_i	Unit inspection duration
τ_r	Unit rectification duration
u_p	Production rate
u_i	Inspection rate
u_r	Rectification rate
u_{max}	Maximum production rate
u_{TP}	Total production rate
u_{TP}^{max}	Maximum total production rate
C^+	Inventory holding cost/units/time unit
C^-	Backlog cost/units/time unit
C_{ins}	Inspection cost
C_{rec}	Rectification cost
C_{def}	Cost of selling-accepting a defective item
C_{pro}	Production cost
C_r	Repair cost
C_m	Preventive maintenance cost
$\beta(\cdot)$	Proportion of defective ítems
$f(\cdot)$	Fraction of inspected products$(0 \le f(\cdot) \le 1)$
Ω_w	Major maintenance policy
n_{max}	Maximum number of repairs considered in the deterioration process
n_p	Accumulated number of repairs that trigger preventive maintenance
AOQ	Average outgoing quality
$AOQL$	Average outgoing quality limit
$AOQL_{max}$	Maximum accepted level of the average outgoing quality limit
$Z_p(n)$	Production thresholds

4.2. Problem description

This paper deals with the analysis of a single-unit manufacturing system subject to deterioration. The machine satisfies a constant demand. However, the machine presented in Fig. 1, experiences random events such as failures and repairs. Thus, given that the production system is unreliable, buffer stock is needed as protection against backlog, during the periods of time where the system is not available because of its disruptions. Furthermore, unsatisfied demand is backlogged with a penalty cost. In response to each failure event, a minimal repair can be conducted, which returns the machine to an *as-bad-as-old* conditions. The machine is subject to a continuous deterioration process which leads to an increase of the defective rate. In this paper, we focus on the case where a quality sampling plan is implemented to ensure a certain average of outgoing quality limit $AOQL_{max}$, required by customers. More specifically, the proposed quality control policy, implies that a sampling fraction of produced items is inspected before being transferred to the inventory stock. Once defective items are identified upon inspection, they are rectified prior to moving them to the inventory stock. Depending on the proportion of defectives found in the inspection, the decision maker can decide to immediately initiate a preventive maintenance. Such maintenance option enables us to completely mitigate the effects of deterioration on the machine and restore its performance to brand new conditions. The durations of the minimal repair and the preventive maintenance are stochastic, and given the set of disturbances that could appear during production, shortages may occur. Thus, a make-to-stock production strategy is needed with the aim to provide protection against possible uncertainties in production, quality control and maintenance. The objective of the model is to jointly determine the production rate, the fraction of production inspected and the major maintenance rate that minimize the total incurred cost. The determined control parameters satisfy the quality constraint required by customers, denoted in this case by $AOQL_{max}$. The total cost includes inventory, backlog, inspection, rectification, repair, preventive maintenance and defectives costs. The optimal solution must ensure that final customers are protected with a constraint on the outgoing quality of items that they receive.

4.3. Assumptions

The model developed in this paper is based on the following assumptions:

(1) The demand rate is known and constant during all the time period.
(2) The deterioration process negatively influences product quality.
(3) The level of deterioration of the machine is defined by the number of repairs.
(4) At failure, a minimal repair is conducted, leaving the machine in *as-bad-as-old*, (ABAO) conditions.
(5) The preventive maintenance implies a perfect repair that restores the machine to *as-good-as new,* (AGAN) conditions.
(6) The totality of the defective units detected in the inspection are rectified before being shipped to the final customer.

We have used these assumptions to have a better understanding of the impact of quality deterioration in the joint determination of production, quality inspection and maintenance strategies, with the aim to extent existing models.

5. Model formulation and proposed control policies

As illustrated in Fig. 1, the manufacturing facility is unreliable, and so the mode of the machine can be described by a stochastic process $\alpha(t)$ with value in $\Omega\{1, 2, 3\}$. More specifically, the machine is operational when $\alpha(t) = 1$, and down when $\alpha(t) = 2$, where a minimal repair is conducted. When $\alpha(t) = 3$, a preventive maintenance is conducted, which implies a perfect repair that restores the system to *as-good-as-new-conditions*.

Given that the manufacturing system is subject to deterioration, our model seeks to identify the impact of such deterioration process on product quality and incorporate the effects of quality-deterioration in the joint control strategy (Colledani & Tolio, 2011). Further at considering the fact that maintenance strategies can be classified according to the degree to which the operating conditions of the machine is restored by maintenance. We consider the quality-deterioration modeling with the use of worse repairs, which implies that the machine is in worse operating condition after a worse repair due to usage, aging and imperfectness of repairs, etc., (Pham & Wang, 1996) and (Wang, 2002). Furthermore, given that in this domain the number of repairs is commonly used as indicator of the level of deterioration of the machine, it serves us to define a failures-deterioration relationship, as in Lam, Zhu, Chan, and Liu (2004). Our formulation implies that repairs have a negative impact of product quality based on the relationships between failures-deterioration and deterioration-quality, leading then to define a failures-quality association as indicated in the following expression:

$$\beta(n) = b_0 + b_1 \cdot \left[\frac{n}{n_{max}}\right]^r \tag{1}$$

where b_0 and b_1 are given constants, n_{max} is the upper limit for the number of repairs, considered in the deterioration process, n is the current number of repairs and r is an adjustment parameter that serves to improve the fit of Eq. (1) to a particular set of data. Similar expressions to Eq. (1) have been successfully used in several studies to model the impact of deterioration as in Dehayem-Nodem et al. (2011a). We need data from historical production to obtain the systems processing capacity and product quality in function of its level of deterioration. Then for given b_0 and n_{max}, factors b_1 and r can be derived from this data through estimation methods, such as the maximum

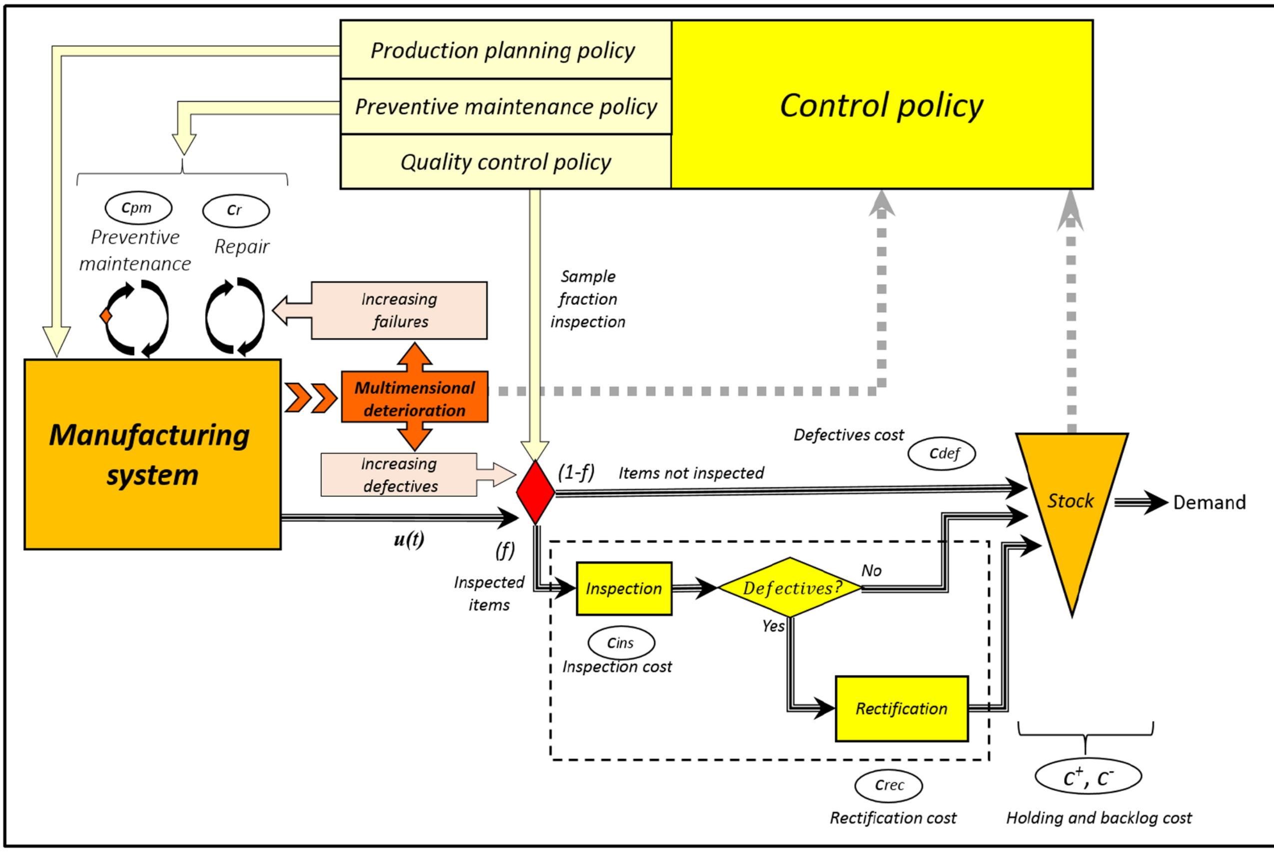
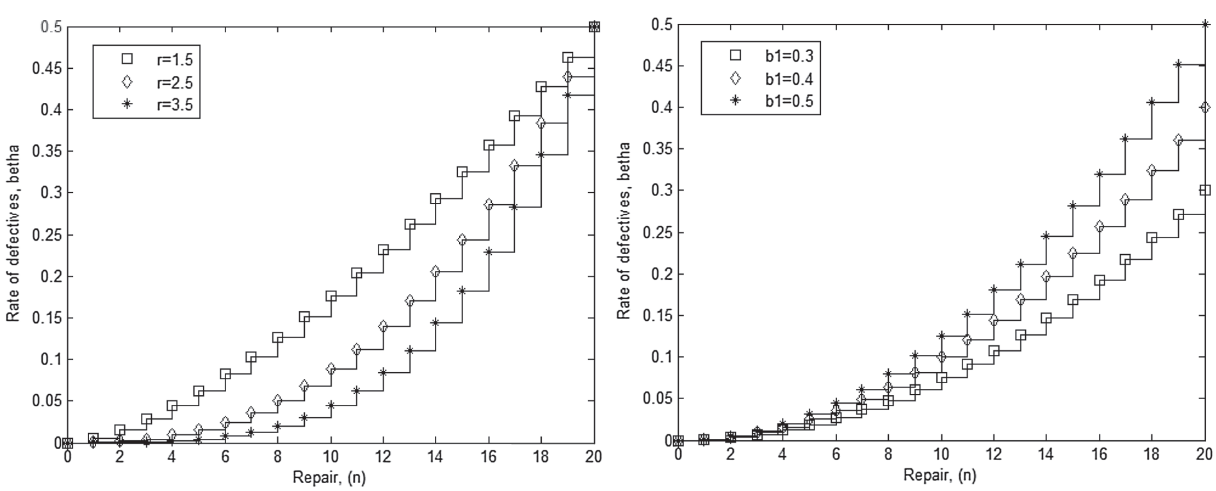


Fig. 1. Block diagram of the proposed production system.



a) Effect of the parameter r b) Effect of the parameter b_1

Fig. 2. Trend of the rate of defectives, for $b_0 = 0$.

likelihood and least squares, such as in Lu et al. (2016). We present in Fig. 2 the effect of the variation of the parameters r and b_1 on the increase of the rate of defectives when we set $b_0 = 0$ and $n_{max} = 20$.

In Fig. 2 we note that the rate of defectives increases as the production system experiences more repairs. Furthermore, defectives units that are not defected in the inspection will reach the final customer at a rate defined by the *average of outgoing quality, AOQ* as follows:

$$AOQ(n) = (1 - f(n))\cdot\beta(n) \tag{2}$$

where $AOQ(n)$ denotes the amount of defectives observed by the final customer and $f(n)$ is the fraction of inspected products. Additionally, it is a common practice in this domain that customers demand a certain quality level, and so we must ensure that the average outgoing quality limit ($AOQL$), defined as the maximum value observed for $AOQ(\cdot)$, does not surpass the limit required by customers, $AOQL_{max}$. In this case the $AOQL$ is calculated as follows:

$$AOQL = \max_{\substack{0\le\beta(n)\le1\\ 0\le f(n)\le1}} \{AOQ(n)\}, \quad n = 0, 1, \cdots, N \tag{3}$$

In this context, the decision-maker must select the optimal combination of preventive maintenance and inspection such that the $AOQL$ does not exceed the maximum limit $AOQL_{max}$ required by customers, impliying that $AOQL \leq AOQL_{max}$.

At considering the quality control activities, it is expected a reduction in the total production rate of the production unit, since we assume that inspection and rectification activities require a non-negligible amount of time for their conduction. In order to incorporate in our formulation the delay caused by inspection and rectification activities, let us first define that the inverse of the production rate u_p denotes the production time as follows:

$$\tau_p = \frac{1}{u_p} \tag{4}$$

Regarding the delay caused by inspection activities, we incorporate in the model the fact that as more units are inspected, more time is needed to conduct such inspection. Then to model this condition we conjecture that the inspection timeτ_i, depends on the sampling fraction $f(n)$conducted as follows:

$$\tau_i = \frac{f(n)}{u_i} \tag{5}$$

where u_i refers to the inspection rate. Eq. (5) implies that the delay of inspection increases, as more units are being inspected and as a consequence the production rate of the unit declines. Furthermore, we model the delay of rectification activities, based on the consideration that the rectification time τ_r is non-negligible and depends on two parameters, namely, the fraction of sampling inspection $f(n)$ and the rate of defectives $\beta(n)$, as indicated by the following expression:

$$\tau_r = \frac{f(n)\cdot\beta(n)}{u_r} \tag{6}$$

where u_r is the rectification rate. Eq. (6) denotes that when the fraction of sampling inspection and the rate of defectives increase, more time is necessary to complete the rectification, and this leads to the reduction of the production capacity. Summing up, Eqs. (4)–(6) enable to model the effect of the inspection and rectification delay on the total production rate in order to determine a feasible production plan. In this case the total production rate u_{TP} is influenced by inspection and rectification activities as follows:

$$u_{TP} = \frac{1}{\tau_p + \tau_i + \tau_r} \tag{7}$$

With $0 \leq u_{TP} \leq u_{TP}^{max}$, where u_{TP}^{max} is the maximum total production rate. Eq. (7) models the progressive reduction in the production rate of the unit, since the delay for inspection and rectification activities increases with the level of deterioration of the system. At assuming that a given rate of non-conforming units will reach the final customers. Thus, the dynamics of the stock level are described by a differential equation, as in the papers of Akella and Kumar (1986), Boukas and Haurie (1990), Polostki, Kenné, and Gharbi (2019):

$$\frac{dx(t)}{dt} = u_{TP}(t) - \frac{d}{1 - AOQ(n)}, \qquad x(0) = x_0 \tag{8}$$

where x_0 is the initial inventory level, $x(t)$ is the inventory at time t, $u_{TP}(t)$ is the total production rate at time t and d is the market demand rate.

5.1. Quality control policy

The quality control strategy proposed in this paper consists of a derivation of a continuous sampling plan proposed by Bouslah et al. (2018) that randomly inspects a fraction $f(\cdot)$ of products with$0 \leq f(\cdot) \leq 1$. However, in contrast of assuming a constant fraction of inspection as in Bouslah et al. (2018), in this paper we conjecture that such fraction $f(\cdot)$ is dynamic and have to be continuously adjusted in function of the level of deterioration of the machine. The sampling plan must be dynamic since we aim in this paper to incorporate the effects of a degrading process with continuous deterioration of part quality (Colledani & Tolio, 2011). Further, it is evident that in the context of quality deterioration, more defectives are produced as the machine deteriorates. Thus as a countermeasure, more units must be inspected as the machine wears, since most of the sampling plan methods increases the sampling fraction $f(\cdot)$ as the rate of defectives increases (Montgomery, 2016). Hence, to facilitate matters at modeling these observations, we can assume that the quality sampling policy is denoted by the following equation:

$$f(n) = f_0 + f_1\left(\frac{n}{n_{max}}\right)^r \tag{9}$$

where f_0 is the fraction of inspected products at initial conditions, f_1 is the maximum limit considered for the sampling fraction and r is a positive constant. Eq. (9) determines the fraction of sampling inspection for each number of repair. The trend of the sampling fraction $f(\cdot)$ for different values of parameters r and f_1 is similar to the trend of $\beta(\cdot)$ illustrated in Fig. 2. A significant advantage of Eq. (9) is that it allows us to model the progressive increase in the sampling fraction as the production unit deteriorates in a not involved manner. Expressions similar to Eq. (9) have been successfully applied to model progressive adjustment in function of the level of deterioration of the machine in this domain of optimal control, as in the papers of Rivera-Gómez et al. (2013) and Dehayem-Nodem, Kenné, and Gharbi (2011b).

5.2. Production-inventory control policy

The proposed production policy is based on the findings of Hlioui, Gharbi, and Hajji (2015a,2015b) and Rivera-Gómez et al. (2013). For the same class of unreliable production systems in a stochastic dynamic context where the machine is facing defective production, Hlioui et al. (2015a,2015b) determined the production rate, the sequence of supply orders and the quality inspection policy consisting on the design of a single acceptance-sampling plan. By considering an imperfect production system with defective production, Hlioui et al. (2015a,2015b) noted that the production policy is effectively controlled by a *Modified Hedging Point Policy* that takes into account the fact that non-conforming units my pass inspection and reach the final customer by an amount denoted by AOQ. Then in the case of imperfect production, we incorporate in our production control rule the amount AOQ of defectives at adjusting the demand rate by $d/(1 - AOQ(n))$ to compensate for the presence of defective units. Furthermore, the final product inventory should be maintained at an excess level in order to face capacity shortages and mitigate the presence of defectives units. However, according to the findings of Rivera-Gómez et al. (2013), at considering the cumulative effects of the deterioration process, such inventory level $Z_p(n)$ must be progressively adjusted to provide further protection against degradation, then it should increase in function of the level of deterioration of the machine. Consequently, in the context of imperfect production and deterioration, a more appropriate production control policy is as follows:

$$u_p(1, x, n) = \begin{cases} u_{TP}^{max} & \text{if } \; x(t) < Z_p(n) \\ d/(1 - AOQ(n)) & \text{if } \; x(t) = Z_p(n) \\ 0 & \text{if } \; x(t) > Z_p(n) \end{cases} \tag{10}$$

where $x(t)$ refers to the inventory level at time$t$ at mode 1 and $Z_p(n)$ is the function that defines the optimal production threshold based on the level of deterioration of the machine.

5.3. Preventive maintenance policy

With respect to the maintenance policy, the machine is submitted to a deterioration-based major maintenance policy. In this case, every time that a repair is conducted, the indicator of the number of repairs n increases. Then we use the number of repairs n, as an indicator of the level of deterioration of the production unit. Thus, the machine is send to preventive maintenance, upon reaching a critical level of deterioration as in Dehayem-Nodem et al. (2011a, 2011b). In order to facilitate the characterization of the preventive maintenance policy, we define Ω_w as a binary function with value 1, if a preventive maintenance is conducted at time t, and 0 if not. For the conduction of preventive maintenance, the number of repairs $n$must attain the critical value n_p^*, and the inventory level must be $x(\cdot) \geq 0$ to avoid further backlog. Thus, the preventive maintenance policy is given by:

$$\Omega_w(1, x, n) = \begin{cases} 1 & if \ \ n(t) \geq n_p^* \ and \ x(\cdot) \geq 0 \\ 0 & otherwise \end{cases} \tag{11}$$

where n_p^* is the critical number of repairs that triggers the conduction of preventive maintenance.

5.4. Optimization of policy parameters

The primary focus of our model is to determine a combination of control parameters ($Z_p^*, f_0^*, f_1^*, r, n_p^*$) that minimizes a key output performance measure of direct economic importance, denoted in this case by the expected average total cost $E\bar{T}C(\cdot)$ and at the same time satisfy the $AOQL$ constraint required by customers. Given that in our case we are interested in the behavior of the system in the *long-run*, then we need several indicators to calculate the key output performance measure $E\bar{T}C(\cdot)$. In our case, the expected average total cost $E\bar{T}C(\cdot)$ consists of the sum of three components, the expected average of the inventory-holding and backlog costs $\bar{IC}(T)$, the expected average of the quality control cost $\bar{QC}(T)$ and the expected average of the maintenance costs $\bar{MC}(T)$.

Regarding the expected average cost of inventory $\bar{IC}(T)$, it comprises three measures of performance that are functions of time, and so *time-persisting statistics* are needed for its calculation, as based in Law (2015). In this case $\bar{IC}(T)$ is given by:

$$\bar{IC}(T), =\frac{1}{T}\cdot \int_0^T (C^+x^+(t) + C^-x^-(t))dt \tag{12}$$

where $x^+ = max(0, x)$, $x^- = max(-x, 0)$, and the constants $C^+$and C^- are used to penalize the inventory and backlog cost, respectively. Where the integrals $\frac{1}{T}\cdot \int_0^T (C^+x^+(t))dt$ and $\frac{1}{T}\cdot \int_0^T (C^-x^-(t))dt$ denote the mean value of the inventory and backlog costs in the time period $[0, T]$.

With respect to the expected average quality cost $\bar{QC}(t)$, it can be determined in the interval $[0, T]$, based on the calculation of *time persistent statistics* of the inspection cost, the rectification cost, the cost of accepting/selling a defective item and the cost of production, as follows:

$$\bar{QC}(T) = \begin{array}{ll} C_{ins}\cdot\frac{1}{T}\cdot \int_0^T (u_p(t)\cdot f(t))dt & \text{(inspection cost)} \\ + C_{rec}\cdot\frac{1}{T}\cdot \int_0^T (u_p(t)\cdot f(t)\cdot\beta(t))dt & \text{(rectification cost)} \\ + C_{def}\cdot\frac{1}{T}\cdot \int_0^T (u_p(t)\cdot AOQ(t))dt & \text{(defectives cost)} \\ + C_{pro}\cdot\frac{1}{T}\cdot \int_0^T (u_p(t))dt & \text{(production cost)} \end{array} \tag{13}$$

where $\frac{1}{T}\cdot \int_0^T (u_p(t)\cdot f(t))dt$, $\frac{1}{T}\cdot \int_0^T (u_p(t)\cdot f(t)\cdot\beta(t))dt$, $\frac{1}{T}\cdot \int_0^T (u_p(t)\cdot AOQ(t))dt$ and $\frac{1}{T}\cdot \int_0^T (u_p(t))dt$lead to define the mean value of the inspection, rectification, defectives and production costs in the interval $[0, T]$, respectively.

The expected average maintenance cost $\bar{MC}(T)$ during the period $[0, T]$ includes the cost of minimal repairs C_r and the cost of preventive maintenance activities C_m and it is given by:

$$\bar{MC}(T) = \frac{C_r\cdot N_r(T) + C_m\cdot N_m(T)}{T} \tag{14}$$

where $N_r(T)$ and $N_m(T)$ are output measures that denote the number of minimal repairs and preventive maintenance conducted in the period$[0, T]$, respectively. Therefore, the optimization problem is to solve the following non-linear constrained stochastic model:

$$\begin{cases} Min \ \ \overline{ETC}(Z_p, f_o, f_1, n_p) = \lim_{T\to\infty}(\overline{IC}(T) + \overline{QC}(T) + \overline{MC}(T)) & \\ \text{Subject to} & \\ \quad AOQL \leq AOQL_{max} & \\ \quad \text{Equations (1)-(8)} & \text{(dynamics of quality and inventory)} \\ \quad \text{Equations (9)-(11)} & \text{(control policy)} \\ \quad 0 \leq f(\cdot) \leq 1 & \\ \quad Z_p(n), f_0, f_1, n_p \geq 0 & \end{cases} \tag{15}$$

The optimization problem (15) should provide the optimal value of the control parameters ($Z_p^*(n), f_0^*, f_1^*, r^*, n_p^*$) that minimize the total incurred cost and that satisfy the quality constraint. Faced with such a control problem, it should be ascertained that given the mathematical difficulties of Eqs. (1)–(14), and the complex interactions between production, quality and maintenance, closed-form solutions for this type of stochastic, non-linear models are not available. Thus, alternative solution methods are needed, in this case we propose a simulation-optimization approach to replace the complex model (15) with an approximated model that can be optimized through non-linear optimization techniques. A simulation-based optimization approach is more suitable in this case to determine a close approximation of the optimal solution, since it is an effective technique to reproduce the set of dynamics and the stochastic behavior of the production system under study. In the next section, we shall delve in more detail of the approach applied.

6. Simulation-optimization approach

Simulation-optimization approaches combine computer simulation with optimization techniques to solve problems that are analytically intractable, such as the model (15) developed in this paper. The solution approach combines mathematical modelling, simulation techniques, design of experiments and response surface methodology with the aim to replace the complex model (15) with an approximated model that we can optimize, leading to the optimal values of the control parameters ($Z_p(n), f_0, f_1, r, n_p$). The solution approach imitates the stochastic and complex behavior of the production system and has successfully solved many complex optimal control problems (see the papers of Lavoie, Gharbi, and Kenné (2010), Gharbi and Kenné (2005), Rivera-Gómez et al. (2013), Bouslah et al. (2016, 2018) and Hlioui et al. (2015a,2015b)). The resolution approach presented in Fig. 3 consists of the following systematic steps:

(1) Mathematical modelling: this step consists in the analytical formulation of the production system under study as detailed in Section 5. This step provides a detailed model of the system dynamics, the objective function to be minimized, the definition of the decision variables and the problem constraint.
(2) Determination of a joint control policy: based on several studies of the literature, a joint control policy is proposed as devised in Eqs. (9)–(11). The control policy is characterized by the control parameters ($Z_p(n), f_0, f_1, r, n_p$) for inventory level, quality sampling plan and maintenance planning. The policy faces random events like failures, repairs and the effects of deterioration.
(3) Simulation model: we transform the mathematical model into a discrete-continuous simulation model following the logic of Section 5. The inputs of such simulation model are defined by the cost parameters (C^+, C^-, C_{ins}, etc.) presented in Table 2, the system's parameters (u_{max}, d, n_{max}, etc.) of Table 3, the dynamics of the mathematical model defined by Eqs. (1)–(8) and the control parameters ($Z_p(n), f_0, f_1, r, n_p$). The purpose of the simulation model is to generate output indicators of the total incurred cost and the limit of the average outgoing quality, AOQL for each value of the inputs. In the next section, we present a detailed description of the proposed simulation model.
(4) Design of experiments: this step uses the outputs of the simulation model to conduct a factorial experimental design (DOE), 3^k and

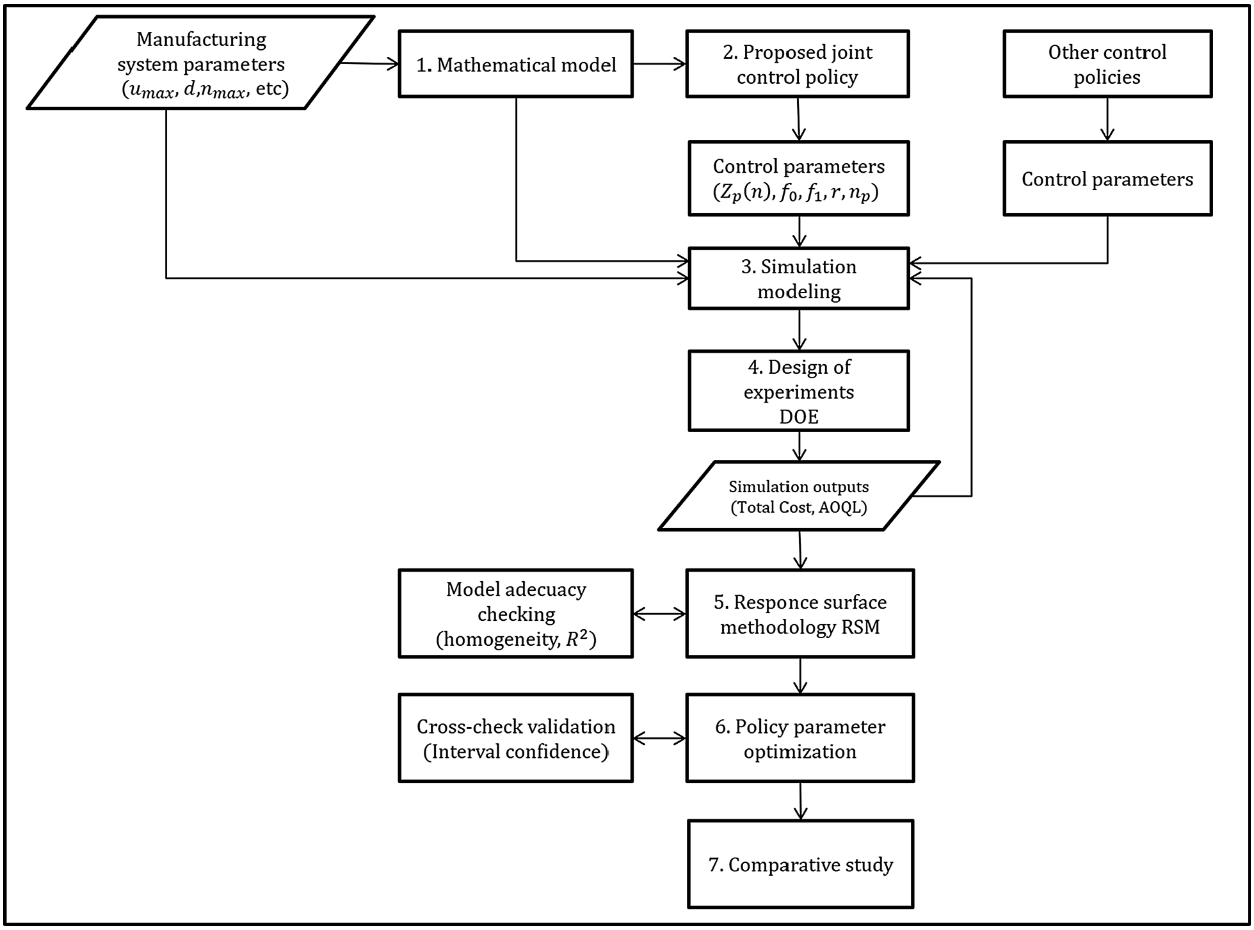


Fig. 3. Simulation-optimization approach.

Table 2
Cost parameters for the numerical example.

Cost:	C^+	C^-	C_{ins}	C_{rec}	C_{def}
Value:	3	150	10	15	185
Cost:	C_r	C_m	C_{pro}		
Value:	200	6000	20		

determine with a minimum number of simulation runs the main factors, interactions and quadratic effects of the control parameters ($Z_p(n)$, f_0, f_1, r, n_p) that significantly affect the simulation model outputs (cost, AOQL) and that must be considered in the optimization step.

(5) Response surface methodology: Once significant factors are identified, we determine second-order regression metamodels, based on the response surface methodology (RSM), for the expected total cost $E\bar{T}C(\cdot)$ and the average outgoing quality limit $AOQL(\cdot)$. The quadratic regression function of the expected total cost $E\bar{T}C(\cdot)$takes the following form:

$$\begin{aligned} E\bar{T}C(\cdot)= & \\ & \gamma_0 + \gamma_1 Z_p(n) + \gamma_2 f_0 + \gamma_3 f_1 + \gamma_4 r + \gamma_5 n_p + \gamma_{11} Z_p^2(n) + \gamma_{12} Z_p(n) f_0 \\ & + \gamma_{13} Z_p(n) f_1 + \gamma_{14} Z_p(n) r + \gamma_{15} Z_p(n) n_p + \gamma_{22} f_0^2 + \gamma_{23} f_0 f_1 + \gamma_{24} f_0 r \\ & + \gamma_{25} f_0 n_p + \gamma_{33} f_1^2 + \gamma_{34} f_1 r + \gamma_{35} f_1 n_p + \gamma_{44} r^2 + \gamma_{45} r n_p + \gamma_{55} n_p^2 + \varepsilon \end{aligned} \tag{16}$$

where γ_0, γ_i, γ_{ii} and γ_{ij}, $(i, j) \in (1, 2, 3, 4, 5)$ represent the regression coefficients and ε is a random error component that incorporates all other sources of variability from uncontrolled factors and general background noise in the process and so forth. The quadratic regression function for the average outgoing quality limit $AOQL(\cdot)$ is:

$$\begin{aligned} AOQL(\cdot)= & \\ & \gamma'_0 + \gamma'_1 Z_p(n) + \gamma'_2 f_0 + \gamma'_3 f_1 + \gamma'_4 r + \gamma'_5 n_p + \gamma'_{11} Z_p^2(n) + \gamma'_{12} Z_p(n) f_0 \\ & + \gamma'_{13} Z_p(n) f_1 + \gamma'_{14} Z_p(n) + \gamma'_{15} Z_p(n) n_p + \gamma'_{22} f_0^2 + \gamma'_{23} f_0 f_1 + \gamma'_{24} f_0 r \\ & + \gamma'_{25} f_0 n_p + \gamma'_{33} f_1^2 + \gamma'_{34} f_1 r + \gamma'_{35} f_1 n_p + \gamma'_{44} r^2 + \gamma'_{45} r n_p + \gamma'_{55} n_p^2 + \varepsilon' \end{aligned} \tag{17}$$

where γ'_0, γ'_i, γ'_{ii} and γ'_{ij}, $(i, j) \in (1, 2, 3)$ are constant parameters and ε'

Table 3
Parameters for the numerical example.

Parameter:	n_{max}	r	b_0	u_r (units/time units)	u_i(units/time units)
Value:	20	2	0	24	60
Parameter:	u_{max}(units/time units)	d(product/time units)	b_1	$AOQL_{max}$	λ_{12} (1/time units)
Value:	14	6	0.45	6%	0.01
Parameter:	λ_{21} (1/time units)	λ_{31} (1/time units)			
Value:	1	5			

is a random error. Adequacy of the regression metamodels (16) and (17) is checked in the region of the optimal solution with the adjusted coefficient of determination R-squared that should be close to one for both expressions. Also a complete examination of residuals is performed to ensure the normality assumption and their homogeneity.

(6) Parameter optimization: Once we obtained the regression models (16) and (17), we replace the unsolvable model (15) with an approximated model (18) that has the advantage that it can be optimized with non-linear constrained optimization techniques, based on the penalty and barrier methods, in this case the MATLAB software was used. Calculation of the optimal solution $(Z_p^*, f_0^*, f_1^*, r^*, n_p^*)$ is given by the following non-linear constrained problem:

(7)

$$\begin{cases} Min \quad \overline{ETC}(Z_p(n), f_o, f_1, r, n_p), \quad (\text{Equation (16)}) \\ \text{Subject to} \\ \quad Equation\ (17) \le AOQL_{max} \\ \quad 0 \le f(\cdot) \le 1 \\ \quad Z_p(n), f_0, f_1, r, n_p \ge 0 \end{cases} \tag{18}$$

Model (18) determines the best values $(Z_p^*(n), f_0^*, f_1^*, r^*, n_p^*)$ which minimize $E\bar{T}C(\cdot)$ and at the same time satisfy the $AOQL(\cdot)$ constraint. Upon the optimization, the optimal solution is cross-checked with extra simulation runs to define a confidence interval for the expected total cost.

(8) Comparative study: The steps 2–6 are conducted in the comparative study section with the aim to highlight the economic advantage of our proposed joint control policy with respect to other representative policies from the literature. It will be shown later in the Comparative study Section that the proposed joint control policy provides better results in terms of the total cost than existing control policies of the literature. The comparative study is conducted in Section 10.

Regression metamodels have been a successful alternative to determine an optimal solution for complex systems as suggested in Gosavi (2014) and Myers, Montgomery, and Anderson-Cook (2009). The sequential procedure of DOE, regression modeling and constrained optimization must be conducted in an appropriate range for the control parameters to fully explore the entire admissible control domain and determine a close approximation of the optimal solution.

7. Simulation model

A discrete/continuous simulation model was developed to reproduce the stochastic behavior of the manufacturing system under analysis. The simulation software ARENA was used to develop such model, which was complemented with code C++ subroutines. The option to develop a discrete/continuous simulation model was selected because this type of models considerably accelerate the simulation execution time, as observed in Lavoie et al. (2010). Further, time economies are necessary given that the adopted simulation-optimization approach requires several simulation runs to define the regression metamodels for the simulation outputs. Additionally, the common random number technique, (Law (2015)) was used to reduce the variability of the model which needs less replications and reduce the size of the confidence interval in the cross-check validation. The simulation model consists of several networks and user routines each if which describes a specific task or event in the system. The differential equation (8) is continuously integrated in the C++ subroutine using the Runge-Kutta-Fehlberg method. The block diagram of this model is presented in Fig. 4, where it is evident the strong inter-relation between the different modules of the model, the block diagram also shows the amount of data that is updated in the model at each time instant.

7.1. Validation of the simulation model

To assess the accuracy of the simulation model, we analyze the evolution of a set of performance indices from a numerical instance. The dynamics of Fig. 5, were obtained when the initial production threshold is set to $Z_p(0) = 20$, $n_p = 12$, $f_0 = 0$, $f_1 = 0.5$, $r = 2$ and n_{max}=15. At examining Fig. 5, we note that at time $t = 0$, (see arrow 1 in Fig. 5d) the production unit is in initial conditions where the effects of deterioration are insignificant, thus it operates at the demand rate $u(t) = d$ to maintain the inventory level at the initial production threshold $Z_p(0) = 20$. After that it experiences several random failures (see arrow 2 in Fig. 5f), and at time $t = 150$, (see arrow 3 in Fig. 5d) it is evident the effects of the deterioration process on the production unit, since it works at rate $u(t) = d/(1 - (1 - AOQ)$ to compensate for the increase of the defectives rates, also we note a progressive increment in the production threshold. Then at time $t = 172$, (see arrow 4 in Fig. 5f), the machine experiences its twelfth failure and so the production threshold reaches its maximum value $Z_p(12) = 25.20$, implying a rate of defectives of $\beta(12) = 0.30$ (see arrow 5 in Fig. 5a). This point indicates that preventive maintenance activities must be conducted (see arrow 6 in Fig. 5e) since the number of failures has reached the critical value $n_p = 12$. Once preventive maintenance is conducted we note a considerable reduction in the inventory level, since preventive maintenance requires a considerable amount of time compared with a minimal repair (see arrow 7 in Fig. 5f). Upon the conduction of preventive maintenance, the production unit is restored to initial conditions, mitigating all the effects of the deterioration process and restoring the production threshold to its initial value $Z_p(0) = 20$. At this point the deterioration cycle reinitiates and the production unit operates at its maximum value $u(t) = u_{max}$ (see arrow 8 in Fig. 5d) to increase the inventory level until the optimum value $Z_p(0) = 20$. From this point, the system dynamics follows a similar deterioration pattern.

A closed examination of Fig. 5 shows that the simulation model developed accurately represents the stochastic behaviour of the production system under analysis, and this serves us to validate our resolution approach.

7.2. Control parameter reduction

The production policy proposed in Section 4, can be effectively characterized at assuming that the production thresholds follow a defined trajectory along the deterioration process. Thus, it is possible to facilitate the definition of such trajectory with an analytical expression based on the results of Mhada et al. (2011). In our case, since the AOQ is the amount of defectives that reaches the final customer, then the trajectory of the production thresholds $Z_p^*(n)$should be adjusted as the value of AOQ increases as proposed by the following expression:

$$Z_p^*(n) = \begin{cases} \frac{Z_{po}}{1 - AOQ(n)} & if \quad 0 \le n \le n_{max} \\ 0 & otherwhise \end{cases} \tag{19}$$

where Z_{po} is the optimal production threshold at initial conditions. In view of Eq. (19), the production threshold will progressively increase in function of the value of the $AOQ(n)$, which increases as the machine deteriorates. The technical advantage of Eq. (19) is that we can define the production policy with just one control parameter Z_{po}. Consequently, Eq. (19) allows us to reduce considerably the number of control parameters.

8. Numerical example

A numerical example of the proposed production system is provided for illustration. Table 2 defines the cost parameters used in the numerical instance. If we assume that at initial conditions the production

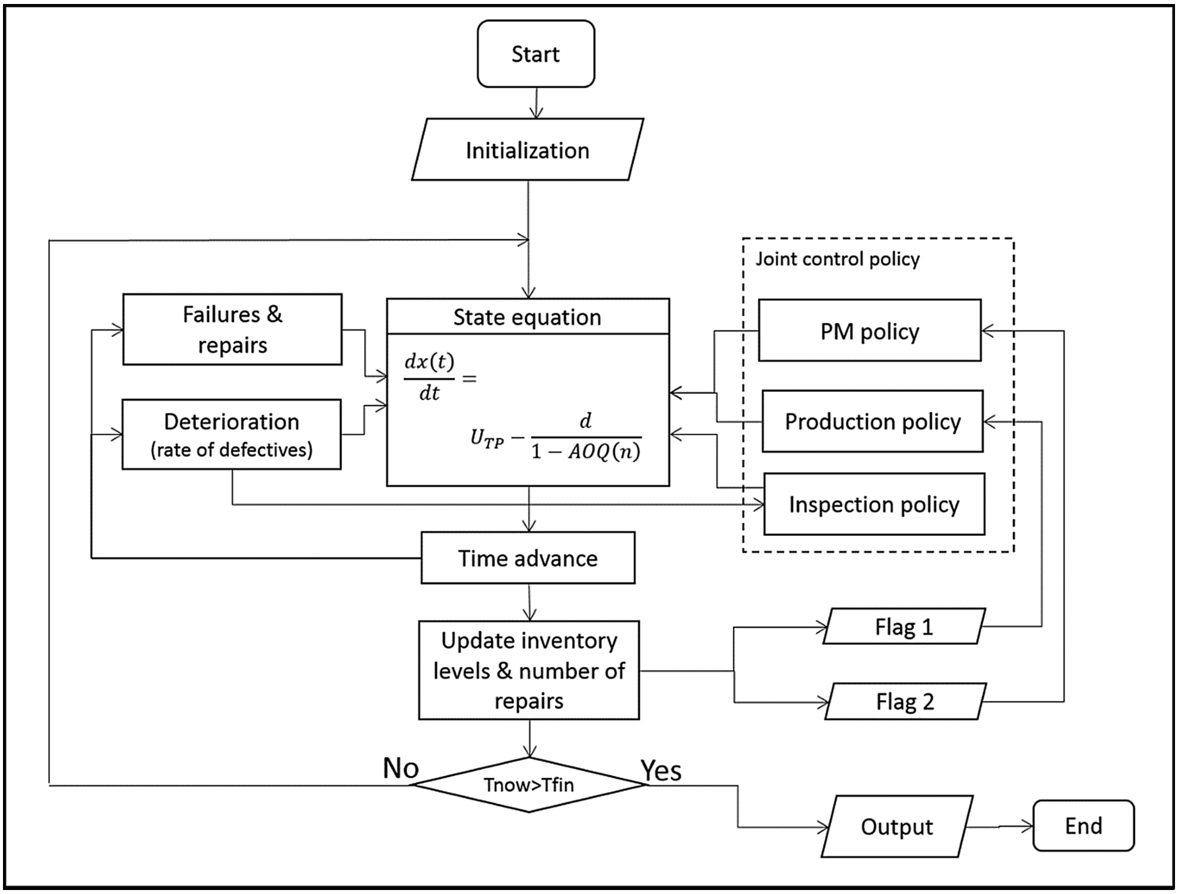


Fig. 4. Simulation block diagram.

system generates a negligible amount of defectives, as presented in Fig. 2, thus $b_0 = 0$. This assumption then leads to define $f_0 = 0$. Moreover, at focusing in the case of increasing defectives, which happens only when $r > 0$. We assume that $r = 2$, for a particular system based on historical quality data. With this assumption we have an increasing fraction inspection as presented in Equation (9). Based on these conjectures, the joint control policy is characterized by only three factors (Z_{po}^*, f_1^*, n_p^*).

The rest of the system parameters are defined as indicated in Table 3.

The statistical analysis of the simulation results was conducted with the statistical software STATGRAPHICS, more details about this analysis are presented in the Appendix A. From such analysis we obtained the following second-order regression equation:

$$E\bar{T}C(Z_{po}, n_p, f_1) = 377.495 - 3.83237\cdot Z_{po} - 4.94921\cdot n_p + 23.5931\cdot f_1 - 0.153023\cdot Z_{po}^2 + 0.0508003\cdot Z_{po}n_p - 0.901778\cdot Z_{po}f_1 + 0.268399\cdot n_p^2 - 2.11406\cdot n_p f_1 + 7.28657\cdot f_1^2 \quad (20)$$

Eq. (20) defines the objective cost function of the approximated model. Such Eq. (20) must be optimized considering the AOQL restriction required by customers. In this case the obtained second-order regression model for the *AOQL* is:

$$AOQL(Z_{po}, n_p, f_1) = 0.011236 - 0.00149174\cdot Z_{po} + 0.0143175\cdot n_p - 0.0168528\cdot f_1 + 0.00000469494\cdot Z_{po}^2 + 0.0000552742\cdot Z_{po}n_p + 0.000552261\cdot Z_{po}f_1 - 0.000242542\cdot n_p^2 - 0.00913539\cdot n_p f_1 + 0.0354286\cdot f_1^2 \quad (21)$$

At considering Eqs. (20) and (21), the optimization problem of Section 5 is presented as follows:

$$\begin{cases} Min \ \text{Equation (20)} \\ \text{Subject to:} \\ \quad Equation\ (21) \le AOQL_{max} \\ \quad 0 \le f(\cdot) \le 1 \\ \quad Z_{po}, n_p, f_1 \ge 0 \end{cases}$$

The cost function (20) is minimized with non-linear programming methods in the MATLAB software to define the optimal values of the control parameters that satisfy the *AOQL* constraint (21). Fig. 6 presents the contour plot of the cost response surface $E\bar{T}C(\cdot)$ on a two-dimensional space. Also, in Fig. 6, contour plots of the *AOQL* constraint are overlaid to show the optimum point.

Solving the optimization problem leads to the optimal solution presented in Table 4. Also a cross-check validation from 50 extra-replications are used to determine the confidence interval presented in Table 4.

The obtained optimal values (Z_{po}^*, n_p^*, f_1^*) are the best parameters to control the joint production, preventive maintenance and quality control policies at a minimum cost.

8.1. *Influence of the* AOQL *constraint*

In this section, we will assess the influence of the *AOQL* restriction on the optimal control policy. In Table 5 we present the optimal solution of the proposed policy for different levels of the *AOQL* restriction. Table 5 also includes *time-persistent statistics* obtained from the

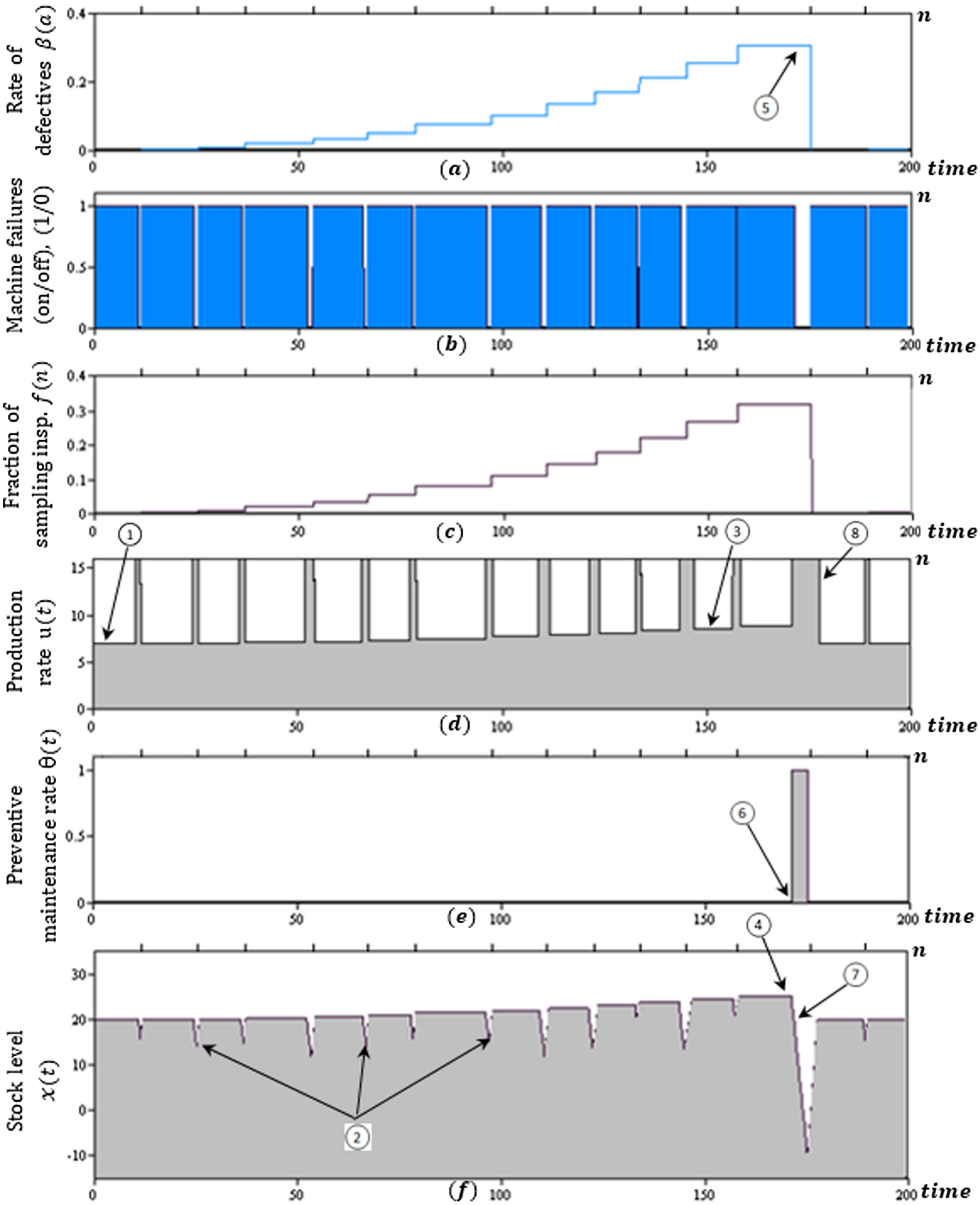


Fig. 5. Dynamics of the simulation model.

simulation model, such as the expected average fraction of production inspected denoted by $\bar{FI}(T)$, which is calculated as follows:

$$\bar{FI}(T) = \frac{1}{T}\cdot\int_0^T f(t)dt \tag{22}$$

Furthermore, the model reports the expected average of outgoing quality $A\bar{O}Q(T)$, which is calculated with the following expression:

$$A\bar{O}Q(T) = \frac{1}{T}\cdot\int_0^T \{(1-f(t))\beta(t)\}dt \tag{23}$$

The simulation model also reports the maximum value of the indicator AOQ, observed in the simulation run, denoted by $AOQL$. From the obtained results, we note that for cases where $AOQL_{max}$ < 6.17% the quality constraint is active. While when $AOQL_{max} \geq$ 6.17%, the constraint is inactive, reporting a minimum cost of $334.38. Furthermore, it is evident that the total expected cost increases as the $AOQL_{max}$ decreases, mainly because expensive preventive maintenance is conducted more frequently to mitigate the effects of deterioration. Faced with a decrease in the $AOQL_{max}$ value, the optimal solution leads to increase the severity of the optimal sampling plan in cases 1–9. In addition, we note a progressive decrement of the inspection efforts, since $\bar{FI}$ steadily decreases, because it is more economical to conduct more frequent preventive maintenance than to inspect more units, since

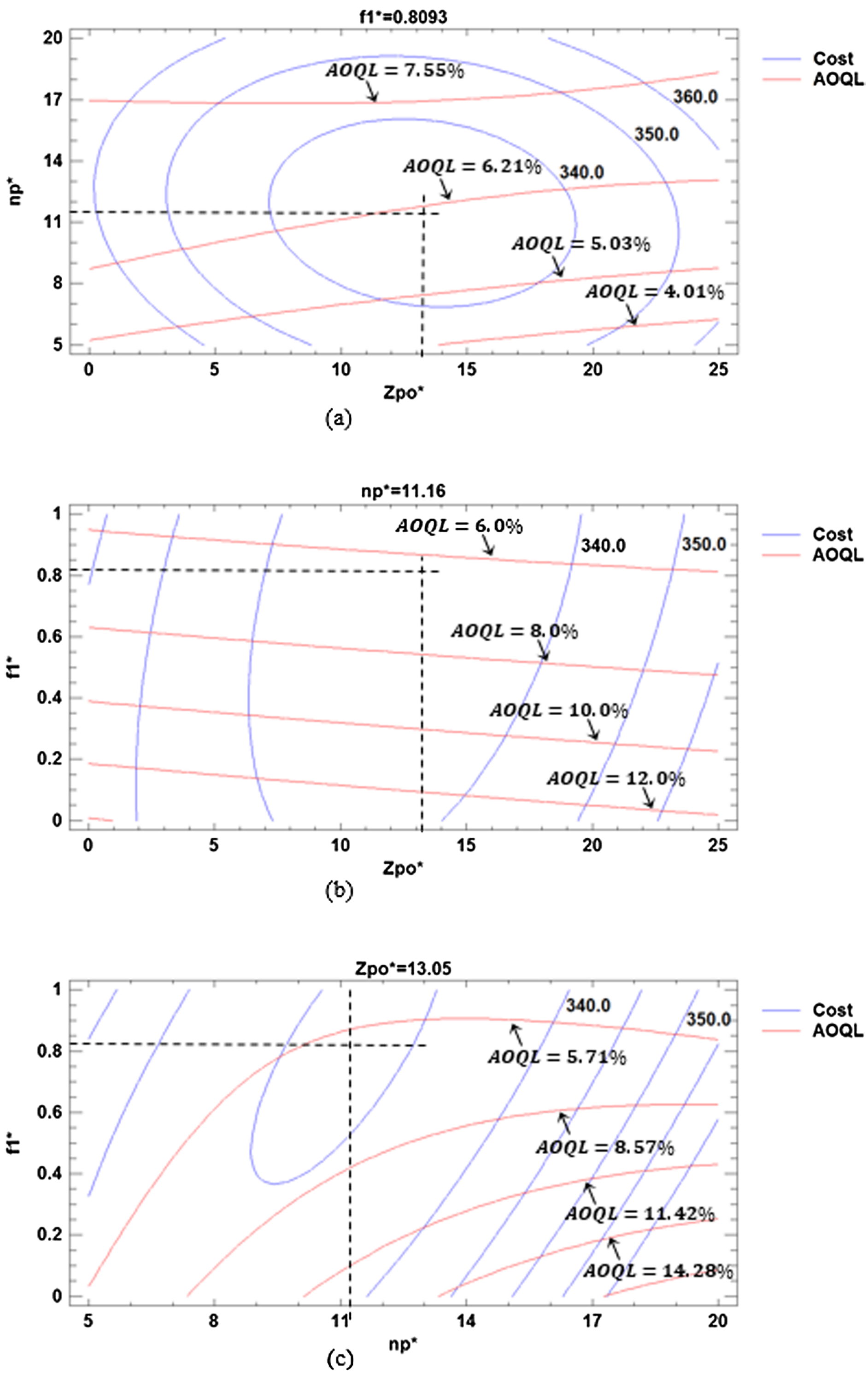


Fig. 6. Cost projections for the optimal control parameters.

Table 4
Optimal solution and crosscheck validation.

	Optimal solution			Optimal cost	
	Z_{po}^*	n_p^*	f_1^*	Total cost $E\bar{T}C(\cdot)$	Confidence interval
Optimal value	13.05	11.16	0.8093	334.38	[332.30, 337.25]

preventive maintenance is a more effective measure to improve quality process. Regarding, the maintenance policy, we note that in all the analyzed cases, the inherent pattern implies that the conduction of preventive maintenance is more frequent as the customer quality requirements becomes more strict, then n_p^* reduces progressively. The reason behind these results is the preventive maintenance efficiency to eliminate defectives. Additionally, we notice that as $AOQL_{max}$ becomes more severe and as preventive maintenance is conducted more frequently, the quality indices improves considerably, and so the average

Table 5
Sensitivity of the $AOQL$ restriction.

Case number	$AOQL_{max}(\%)$	Control parameters			Quality indices			$E\bar{T}C^*(\cdot)(\$)$
		Z_{po}^*	n_p^*	f_1^*	$\bar{FI}(\%)$	$A\bar{O}Q(\%)$	$AOQL(\%)$	
Basic case	**≥6.17**	**13.05**	**11.16**	**0.8093**	**23.38**	**3.70**	**6.17**	**334.38**
1	6	13.06	10.04	0.9295	23.24	3.31	6	334.69
2	5.5	14.16	9.31	0.9999	19.60	3.03	5.5	335.48
3	5	13.73	6.11	0.6182	9.49	2.60	5	337.76
4	4.5	13.68	5.13	0.5056	6.39	2.28	4.5	339.27
5	4	13.65	4.37	0.4226	4.31	1.90	4	340.70
6	3.5	13.63	3.73	0.3539	2.77	1.51	3.5	342.09
7	3	13.62	3.16	0.2937	2.28	1.49	3	343.44
8	2.5	13.61	2.64	0.2396	1.35	1.11	2.5	344.77
9	2	13.60	2.17	0.1900	1.06	1.08	2	346.09

outgoing quantity $A\bar{O}Q$ and the $AOQL$ reduces from cases 1–9. With respect to the production policy, Z_{po}^* increases in cases 1–2, when f_1^* increases, as protection against defectives. However, Z_{po}^* decreases in cases 3–9, as n_p^* reduces since less protection is needed against defectives as preventive maintenance is conducted more frequently.

We complement the sensitivity analysis of the $AOQL$ restriction with the results presented in Fig. 7. The first observation from Fig. 7a, is that as expected the indicator of the expected average fraction of production inspected $\bar{FI}$, is higher for all values of $AOQL_{max}$, when the inspection cost is reduced to $C_{ins} = 7$, this is because, more inspection can be conducted at reducing C_{ins}. However, when the inspection cost increases to $C_{ins} = 12$, we clearly observe that the $\bar{FI}$ indicator is always inferior than in the previous case leading to conduct less inspection for any $AOQL_{max}$ value. This reduction in $\bar{FI}$ is because at increasing c_{ins}, inspection activities are more penalized and thus less conducted.

Additionally, we present in Fig. 7b the trend of the critical number of repairs n_p^* that triggers preventive maintenance in function of the severity of the $AOQL$ restriction. In particular, we observe that when the preventive maintenance cost decreases to $C_m = 5500$, the critical number of repairs n_p^* reduces for all values of the $AOQL_{max}$, this implies that preventive maintenance is conducted earlier to restore the machine to initial conditions and mitigate the effects of deterioration. Note that when the preventive maintenance cost increases to $C_m = 6500$, the conduction of preventive maintenance is delayed, increasing then n_p^* for all the considered values of $AOQL_{max}$.

9. Sensitivity analysis

The idea behind this section is to conduct another set of simulation runs to analyze the sensitivity of the proposed model with respect to the variation of different cost parameters, such as the inventory, backlog, repair, preventive maintenance, inspection, rectification, and defectives costs and several systems parameters. The objective is to have a better comprehension of the behavior of the proposed model and compare the total incurred cost for different system conditions derived from a basic case.

9.1. Influence of the cost parameters

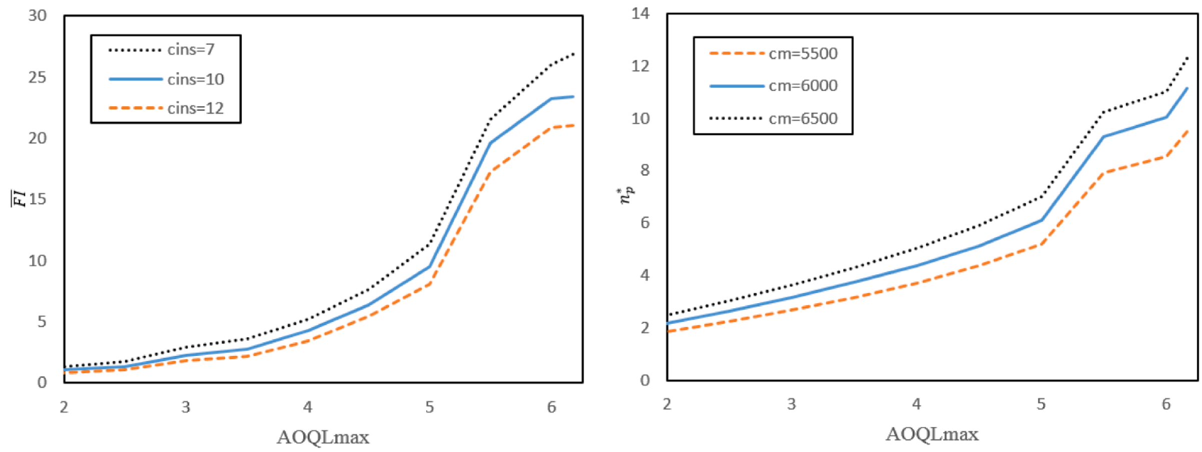
We present 16 different cost configurations derived from a basic case by varying their values above and below from a base of comparison. The obtained results of such sensitivity are presented in Table 6.

The variations of the optimal solution compared to the basic case make sense and can be explained as follows:

- Variation of the inventory cost and backlog cost: When the backlog cost c^- increases (case 13), the production threshold Z_{po}^* increases as a countermeasure to provide better protection against shortages. At increasing c^-, the unit operates more time at its maximum rate, deteriorating more and producing more defectives. Thus, the severity of the sampling plan increases, and $\bar{FI}$ increases as an attempt to enhance customer protection. With more inspection efforts, less defectives reaches the final customer then $A\bar{O}Q$ and $AOQL$ decrease. Additionally, with more inspection efforts, the conduction of preventive maintenance is delayed, increasing then n_p^*, because the system prefers to inspect more units than to pay the high cost of preventive maintenance. The decrease of c^- has the opposite effects (case 12). Regarding the sensitivity of the inventory cost, c^+ (cases 10 and 11), we note that it has the contrary effects that the backlog cost.
- Variation of the repair and preventive maintenance cost: at increasing the preventive maintenance cost c_m (case 17) it is normal to delay this activity, increasing then n_p^*. Moreover, as less preventive maintenance is conducted, the severity of the sampling plan rises to compensate, hence $\bar{FI}$ increases. Also with the delay of preventive maintenance, the production threshold increases to improve the protection against shortages and defectives. The increment of the inspection efforts $\bar{FI}$, leads to reduce the amount of defectives that reach the final customer, then $A\bar{O}Q$ and $AOQL$ decrease. Further, we note that the decrease of c_m produces the opposite effects (case 16). From Table 6, the repair cost c_r, (cases 14 and 15) has inverse effects that the preventive maintenance cost.
- Variation of the production and defectives cost: when c_{pro} increases (case 19), it has the direct effect to reduce the production threshold Z_{po}^* to limit the amount of stock to an essential level. Furthermore, at increasing c_{pro} more frequent preventive maintenance is conducted, reducing n_p^*, to maintain the machine in exceptional conditions. With more frequent preventive maintenance, the sampling plan severity reduces, hence $\bar{FI}$ decreases. Nevertheless, with less inspection efforts, the $A\bar{O}Q$ and $AOQL$ increases. We note the contrary effects when c_{pro} decreases (case 18). Further, we observe that the variation of c_{def} (cases 24 and 25) yields to the inverse effect of the production cost.
- Variation of the inspection and rectification cost: when the inspection cost c_{ins} increases (case 21), it is logical that the inspection efforts reduce, then $\bar{FI}$ decreases. Further, the production threshold Z_{po}^* reduces because the cost of inspection is tied directly with the production rate in the cost function (13), and this limits the stock level to the bare minimum. Also at increasing c_{ins}, preventive maintenance is conducted more frequently as an attempt to improve process quality, thus n_p^* decreases. Nevertheless, with the decrease of $\bar{FI}$, more defectives reaches the final customer, then the system capacity reduces, and $A\bar{O}Q$ and $AOQL$, increase. We observe that a lower inspection cost has the inverse effects (case 20). Moreover, the variation of c_{rec}(cases 22 and 23) has similar effects that the inspection cost.

9.2. Influence of system parameters

The remainder of this section analyzes the influence of a number of



a) Effect of the inspection cost on $\overline{FI}$ b) Effect of the preventive maintenance cost on n_p^*

Fig. 7. Analysis of the variation of $\bar{FI}$ and n_p^* for each value of $AOQL_{max}$.

Table 6
Sensitivity analysis.

Par.	Case number	Value	Control parameters Z_{po}^*	n_p^*	f_1^*	Quality indices $\bar{FI}$(%)	$A\bar{O}Q$(%)	$AOQL$(%)	$E\bar{T}C^*(\cdot)$	Remark
–	**Basic case**	–	13.05	11.16	0.8093	23.38	3.70	6.17	334.38	Based for the comparison
C^+	10	2.2	16.06	11.45	0.9899	27.78	3.29	5.04	322.76	$Z_{po}^*\uparrow, n_p{}^*\uparrow, \bar{FI}\uparrow$
	11	3.5	10.83	10.44	0.5813	15.07	4.03	7.90	340.34	$Z_{po}^*\downarrow, n_p{}^*\downarrow, \bar{FI}\downarrow$
C^-	12	120	10.21	9.46	0.4566	10.51	3.88	9.35	330.47	$Z_{po}^*\downarrow, n_p{}^*\downarrow, \bar{FI}\downarrow$
	13	185	15.04	11.81	0.9410	26.98	3.45	5.30	337.50	$Z_{po}^*\uparrow, n_p{}^*\uparrow, \bar{FI}\uparrow$
C_r	14	50	13.21	11.75	0.8978	25.33	3.51	5.56	321.36	$Z_{po}^*\uparrow, n_p{}^*\uparrow, \bar{FI}\uparrow$
	15	400	12.72	10.11	0.6387	16.46	3.89	7.46	351.59	$Z_{po}^*\downarrow, n_p{}^*\downarrow, \bar{FI}\downarrow$
C_m	16	5500	12.07	8.09	0.3036	6.25	3.72	9.21	329.79	$Z_{po}^*\downarrow, n_p{}^*\downarrow, \bar{FI}\downarrow$
	17	6500	13.25	12.39	0.9412	29.33	3.52	5.30	337.95	$Z_{po}^*\uparrow, n_p{}^*\uparrow, \bar{FI}\uparrow$
C_{pro}	18	16	13.25	12.16	0.9899	30.79	3.38	5.04	312.13	$Z_{po}^*\uparrow, n_p{}^*\uparrow, \bar{FI}\uparrow$
	19	24	12.86	10.03	0.6110	15.78	3.96	7.67	363.63	$Z_{po}^*\downarrow, n_p{}^*\downarrow, \bar{FI}\downarrow$
C_{ins}	20	7	13.30	11.66	0.9235	26.26	3.47	5.41	334.06	$Z_{po}^*\uparrow, n_p{}^*\uparrow, \bar{FI}\uparrow$
	21	12	11.97	9.24	0.3332	7.62	4.04	9.03	335.54	$Z_{po}^*\downarrow, n_p{}^*\downarrow, \bar{FI}\downarrow$
C_{rec}	22	13.7	13.33	11.74	0.9377	26.70	3.44	5.32	334.11	$Z_{po}^*\uparrow, n_p{}^*\uparrow, \bar{FI}\uparrow$
	23	17	12.73	10.53	0.6623	17.22	3.87	7.28	334.68	$Z_{po}^*\downarrow, n_p{}^*\downarrow, \bar{FI}\downarrow$
C_{def}	24	170	12.23	10.72	0.5078	12.89	4.09	8.45	330.25	$Z_{po}^*\downarrow, n_p{}^*\downarrow, \bar{FI}\downarrow$
	25	190	13.32	11.53	0.9230	26.31	3.48	5.41	335.10	$Z_{po}^*\uparrow, n_p{}^*\uparrow, \bar{FI}\uparrow$

system parameters on the optimal control factors (Z_{po}^*, n_p^*, f_1^*). In particular, we analyze the impact of the adjustment parameter r, which serves to modify the pace of generation of defectives units, also we study the sensitivity of the inspection u_i and rectification u_r rates. We complement the discussion with the effect of the rate of defectives q_{12}. Table 7 presents eight different system configurations that allow us to analyze the effect of the variation of parameters (i.e. r, u_i, u_r and q_{12}).

The interpretation of the sensitivity of the parameters is as follows:

- Variation of the quality deterioration rate: Recall that the adjustment parameter r serves to modify the pace of generation of defectives. In particular, when $r < 1$, the system accelerates the

Table 7
Sensitivity of system parameters.

Par.	Case number	Value	Control parameters Z_{po}^*	n_p^*	f_1^*	Quality indices $\bar{FI}$(%)	$A\bar{O}Q$(%)	$AOQL$(%)	$E\bar{T}C^*(\cdot)$	Remark
–	**Basic case**	–	13.05	11.16	0.8093	23.38	3.70	6.17	334.38	Based for the comparison
r	26	1.7	13.30	11.75	0.8538	25.73	4.37	6.58	340.06	$Z_{po}^*\uparrow, n_p{}^*\uparrow, \bar{FI}\uparrow$
	27	2.4	12.35	10.34	0.7897	16.99	3.11	6.05	323.47	$Z_{po}^*\downarrow, n_p{}^*\downarrow, \bar{FI}\downarrow$
u_i	28	15	13.31	9.40	0.4042	9.15	3.99	8.59	326.10	$Z_{po}^*\uparrow, n_p{}^*\downarrow, \bar{FI}\downarrow$
	29	22	12.97	11.44	0.8375	24.04	3.64	5.96	334.78	$Z_{po}^*\downarrow, n_p{}^*\uparrow, \bar{FI}\uparrow$
u_r	30	7	13.16	9.59	0.4625	10.73	3.92	9.29	331.02	$Z_{po}^*\uparrow, n_p{}^*\downarrow, \bar{FI}\downarrow$
	31	12	12.80	11.93	0.8493	24.38	3.61	5.87	335.05	$Z_{po}^*\downarrow, n_p{}^*\uparrow, \bar{FI}\uparrow$

Table 8
Comparative study.

Description	Control parameters			Quality indices			$E\bar{T}C^*(\cdot)$	Cost differenceΔ-Cost (%)
	Z_{po}^*	n_p^*	f_1^*	$\bar{F}I$(%)	$A\bar{O}Q$(%)	$AOQL$(%)		
Policy-I	**13.05**	**11.16**	**0.8093**	**23.38**	**3.70**	**6.17**	**334.38**	–
Policy-II	18.79	15.74	–	100.0	0.00	0.00	392.83	+17.48%
Policy-III	14.43	12.60	0.2983	29.83	4.39	10.47	380.213	+13.70%
Policy-IV	14.45	20	0.7551	45.34	4.62	6.61	72.88	+11.51%

generation of defectives and vice-versa. From Table 7, when r increases (case 27), the deterioration rate decreases and the model reacts by reducing the production threshold Z_{po}^* because the system produces less defectives and so the system capacity increases and there is less need for shortage protection. With less defectives, the plan severity reduces (then $\bar{F}I$ decreases), and so the system can afford more frequent preventive maintenance, reducing n_p^*. Further, the conduction of more frequent preventive maintenance and the presence of less defectives lead to improve the quality indices, thus $A\bar{O}Q$ and $AOQL$ reduce. The decrease of r produces the opposite effects (case 26).

- Variation of the inspection and rectification rate: the increase of the inspection rate u_i (case 29) reduces the inspection time τ_i as denoted in Equation (5), and with the increment of u_i the system increases its production capacity. Hence, the production threshold Z_{po}^* reduces because the system can produce and inspect items at a faster pace. Moreover, with the reduction of Z_{po}^*, the machine deteriorates less, hence preventive maintenance is delayed, increasing then n_p^*. Additionally, at delaying preventive maintenance, more inspection is conducted, thus $\bar{F}I$ increases. In addition, with more inspection less defectives reaches the customer, then $A\bar{O}Q$ and $AOQL$ decrease. From the results of Table 7, we note that the decrease of the inspection rate generates the contrary effects (case 28). Further, the variation of the rectification rate u_r (cases 30 and 31) has similar effects as the variation of the inspection rate.

10. Comparative study

Based on the analysis of the set of papers presented in literature review, in this section we conduct a comparison study to highlight the economic advantages of our proposed control policy. In particular, we observe that current literature dissociates production, quality inspection and maintenance decisions, and most of the researchers disregard the importance of a dynamic inspection policy that must be adjusted in function of the level of deterioration of the machine. We compare the performance of the prosed control policy, which we denote as Policy-I, to the most representatives policies from the literature. The other policies considered in the comparison are described as follows:

- Policy-II: this policy is derived from the results of the paper of Rivera-Gómez et al. (2013), where the quality sampling control is not considered in the optimization, thus the control strategy consists of 100% inspection of all the items produced instead of using a sampling inspection. This Policy II can be viewed as a simplified version of Policy-I, without the optimization of the optimal sampling fraction.
- Policy III: this policy is based on the results of Bouslah et al. (2018), where the optimal production threshold and the optimal sampling fraction inspection are constants and do not evolve in function of the deterioration of the machine. For this Policy III, Z_p^* and f^* remain constants during the simulation.
- Policy IV: this policy derives from Policy-I, with the difference that the determination of preventive maintenance is not part of the optimization. In this Policy-IV, preventive maintenance is conducted when the machine reaches the maximum allowed number of failures n_{max} and the optimization only determines the optimal production threshold and the optimal sampling fraction inspection.

Table 8 presents the total incurred cost and complementary performance indices, obtained by using the same basic case data for the considered policies.

The interpretation of the obtained results is as follows:

- Effect of Policy-II: this policy ensures the delivery of defect-free products through the 100% inspection of the items produced. Thus, under this policy, the quality indices $A\bar{O}Q$ and $AOQL$ are zero because all produced units are inspected. However, since in Policy-II inspection requires more time, the production threshold Z_{po}^* increases to ensure demand satisfaction. In addition, preventive maintenance is delayed because the quality indices $A\bar{O}Q$ and $AOQL$ improve with 100% inspection. Regarding the total cost, Policy-II reported a 17.48% higher cost than Policy-I. The observed difference is due to the extra unnecessary inspection conducted by Policy-II during the beginning of the deterioration process, when the process quality is excellent, and so conducting 100% inspection unnecessarily increases the total cost.
- Effect of Policy-III: From the results of Table 8, we note that 29.83% of items will be inspected in the long-term. However, the quality indices are negatively affected, since $A\bar{O}Q$ and $AOQL$ increases, mainly because the sampling fraction is not adjusted in function of the level of deterioration. Furthermore, the parameter Z_{po}^* increases as protection against defectives, because in Policy-III, Z_{po}^* is not dynamic, it is constant during the simulation. With fixed Z_{po}^* and f^*, preventive maintenance is delayed to avoid disrupting the machine from production. In Policy-III the total cost increases, reporting a difference of 13.70% compared to Policy-I, because in Policy-III Z_p^* and f^* are not adjusted progressively as the machine deteriorates.
- Effect of Policy-IV: In this policy, preventive maintenance is conducted when the machine reaches the limit n_{max} of failures. Thus, it is expected that the total cost of Policy-IV is considerably higher than the cost of Policy-I, with an observed difference of 11.51% because at conducing less preventive maintenance the machine reaches higher levels of deterioration before restoration, reflected in more defectives. Hence, with less preventive maintenance the inspection efforts $\bar{F}I$ increases as a countermeasure to detect more defectives. Nevertheless such preventive maintenance delay has a negative impact on the quality indices $A\bar{O}Q$ and $AOQL$, which increase. Further, with the presence of more defectives, the production threshold Z_{po}^*increases as protection. The increment of the amount of defectives and inspection explain the increment of the total cost of Policy-IV.

11. Conclusion

Traditionally the fields of production, quality and maintenance planning has been treated as separate problems in the literature despite their evident interaction. Currently, it exists a limited number of paper that address these three key functions simultaneously in an integrated model. Nowadays, the design of sampling plans has evolved from considering only quality requirements with no economic consideration and disregarding the influence on production and maintenance planning to a more integrated view that takes into account economic repercussions and interactions with production and maintenance aspects. In this paper, we have developed a new integrated model for the joint optimization of production, preventive maintenance and quality sampling plan for a quality deteriorating production system considering an outgoing quality constraint. The proposed model contribute to the domain of optimal control of production systems at considering the effect of a deterioration process on the determination of the optimal production, quality and preventive maintenance control parameters. The proposed control parameters for the inventory level and the quality-sampling fraction are not constant as previously considered in the literature, they are dynamic in our formulation and their values vary in function of the level of deterioration of the production system. In the sensitivity analysis, we analyzed the effect of an extensive number of cost and system parameters that highlight the strong interaction between production, quality control and preventive maintenance strategies. In the comparative study, we observed significant cost economies, if the control parameters are dynamic and evolve in function of the level of deterioration of the machine. In a sense, it can be stated that the obtained results are quite satisfactory and foster further research in this domain. Possible extensions of this work could investigate more complex production systems such as the case of unreliable suppliers where the need to inspect income material are taken into account. Further research can be conducted to study the case where imperfect maintenance have a relationship with the severity of the sampling plan.

CRediT authorship contribution statement

Héctor Rivera-Gómez: Writing - original draft, Software, Validation. **Ali Gharbi:** Conceptualization, Methodology, Supervision, Writing - review & editing. **Jean-Pierre Kenné:** Supervision, Validation, Writing - review & editing. **Oscar Montaño-Arango:** Supervision, Writing - review & editing. **Jose Ramón Corona-Armenta:** Writing - review & editing.

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Appendix A

Simulation runs are conducted according to a complete 3^3 factorial design to screen out a subset of the control factors (Z_{po}^*, f_1^*, n_p^*) that have a significant impact on the response$E\bar{T}C(\cdot)$. For each combination of independent factors (Z_{po}^*, f_1^*, n_p^*), the experimental design is replicated three times implying a total of $(3^3 \times 3) = 81$ simulation runs. We are interested to take significant factors and build a metamodel of how the simulation model transforms a particular set of input-factor values into the output response, $E\bar{T}C(\cdot)$ and the $AOQL$ indicator. Based on off-line simulation runs we define the minimum and maximum values of the factors (Z_{po}^*, f_1^*, n_p^*) as presented in Table A1.

Without loss of generality, we set the limit of the number of failures as $n_{max} = 20$. The simulation run length is set to 100,000 units of time to ensure steady state conditions. The simulation results are handled with the statistical software STATGRAPHICS in order to obtained an analysis of variance (ANOVA) and determine a metamodel for the expected average total cost $E\bar{T}C(\cdot)$. The ANOVA analysis for the total cost is presented in Table A2.

The adjusted R-squared coefficient of the data of Table A2 is $R^2 = 94.40\%$, implying that about 95% of the observed variability in the total cost $E\bar{T}C(\cdot)$ is explained by the second-order model. From Table A2 we note that all the main factor and most of the interactions are significant with a P-value≤5%. Regarding the AOQL constrained we obtained the following ANOVA Table A3.

The obtained R-squared coefficient for the AOQL constraint is equal to 98.52% implying a good fit for the data.

Table A1
Range for the independent variables.

Factor	Low level	High level	Description
Z_{po}	5	25	Initial production threshold
n_p	5	20	Number of failure to restore the system
f_1	0.05	0.95	Maximum sampling fraction inspection

Table A2
The ANOVA Table for the total cost.

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
A:Zpo*	5029.0	1	5029.0	230.53	0.0000
B:np*	6459.01	1	6459.01	296.08	0.0000
C:f1*	959.202	1	959.202	43.97	0.0000
AA	5643.9	1	5643.9	258.72	0.0000
AB	463.892	1	463.892	21.26	0.0000
AC	462.828	1	462.828	21.22	0.0000
BB	4420.58	1	4420.58	202.64	0.0000
BC	1952.09	1	1952.09	89.48	0.0000
CC	17.7124	1	17.7124	0.81	0.3707
blocks	6.44023	2	3.22011	0.15	0.8630
Total error	1505.24	69	21.8151		
Total (corr.)	26919.9	80			

Table A3
The ANOVA Table for the AQOL constraint.

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
A:Zpo*	0.000739178	1	0.000739178	18.49	0.0001
B:np*	0.0620188	1	0.0620188	1551.45	0.0000
C:f1*	0.0803769	1	0.0803769	2010.69	0.0000
AA	1.2708E−7	1	1.2708E−7	0.00	0.9552
AB	0.000549777	1	0.000549777	13.75	0.0004
AC	0.000122026	1	0.000122026	3.05	0.0851
BB	0.00334894	1	0.00334894	83.78	0.0000
BC	0.0362571	1	0.0362571	907.00	0.0000
CC	0.00100575	1	0.00100575	25.16	0.0000
blocks	0.0000218531	2	0.0000109265	0.27	0.7617
Total error	0.00275826	69	0.0000399748		
Total (corr.)	0.187199	80			

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