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An agent-oriented decision support system combining fuzzy clustering and the AHP

Omar López-Ortega *, Marco-Antonio Rosales

Centro de Investigación en Tecnologías de Información y Sistemas (CITIS), Universidad Autónoma del Estado de Hidalgo, Carretera Pachuca – Tulancingo Km. 4.5, C. P. 42084 Pachuca, Hidalgo, Mexico

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ABSTRACT

Decision making is a complex process, particularly when it is carried out by multidisciplinary team. Methods based on the analytical hierarchy process have been widely employed because they provide solid mathematical background. Nevertheless, solutions such as the Aggregation of Individual Judgements (AIJ) and the Aggregation of Individual Priorities (AIP) do not offer sufficient explanatory data in regards with the final decision. We developed an agent-based decision support system (DSS) that employs fuzzy clustering to group individual evaluations and the AHP to reach a final decision. Fuzzy clustering is adequate to determine important pieces of data such as the largest group of evaluations that exist around a centroid value. On the other hand, the MAS paradigm offers capabilities for achieving distributed and asynchronous processing of data. The AHP is used after the individual evaluations are clustered, as if the group were a single evaluator. Altogether, the proposed solution enhances the quality of multi-criteria group decision making.

1. Introduction

As it is suggested in Carmen and French (2003), modern management promotes distributed decision making carried by multidisciplinary teams. Organizations decide to promote group decision making, where experts work together but not necessarily at the same place or time (Soubie & Zaraté, 2005). For example, when it comes to acquire new manufacturing equipment (Rao, 2007), or to select the best personnel, among many alternatives (Metin, in press), not only the opinion of one single person is taken into account. Evaluations from qualified people with different background and perspectives are favored nowadays. Top management must broadcast, to those individuals that will form the decisional group, the evaluation criteria as well as specific data of the alternatives. In turn, the evaluators must judge the alternatives, and top management shall make a final decision based on such judgements.

Thus, decision making refers at selecting, among a finite set of m alternatives, the one that complies best with a finite set of p evaluation criteria. This particular problem has been tackled by Saaty, who developed the well-known Analytic Hierarchy Process (AHP) (Saaty, 1977). Let us suppose, however, that top management decides to gather opinions from p experts. Should the AHP be used as a decision process, a pairwise comparison matrix (PCM) is formed in order to compare the relative importance of the evaluation criteria. Therefore, management will be forced to process z **PCM**'s to determine the group assessments.

To achieve group decision making based on the AHP, three different methods have been proposed. The Aggregation of Individual Judgements (AIJ), and the Aggregation of Individual Priorities (AIP) (Forman & Penitawi, 1998). Also, an optimization method has been proposed by Sun and Greenberg (2006). However, neither of the three mentioned approaches actually provides information on how the group of experts accommodated. For instance, it is not possible to determine how many of the evaluators agree on the resultant priorities. This is so because such techniques are based on geometrical averages.

Hence, to enhance group decision making, we developed a solution based on the combination of Multi-Agent Systems, the fuzzy C-means clustering technique and the Analytic Hierarchy Process. The proposed decision support system (DSS) allows distribution, asynchrony and clusters formation based on fuzzy c-means. Multi-Agent Systems fulfill technological needs related to automating the distribution and processing of large amounts of data. Fuzzy clustering is adequate to determine how many evaluations actually form the group majority. Also, it is established the value around which every single evaluation is close enough to be considered part of the winning cluster. This data is the largest cluster's centroid. Furthermore, it is also possible to determine how compact the clusters are by computing data dispersion. Finally, the AHP is used to reach a final ranking of the alternatives once the experts' evaluations are grouped.

The DSS we present provides the following modules. One module, residing at the management's site, is used to define evaluation

* Corresponding author.
E-mail addresses: lopezo@uaeh.edu.mx, lopezo@uaeh.reduaeh (O. López-Ortega).

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criteria, and broadcast such criteria to the evaluators. They, in turn, possess an evaluation module that helps collecting the judgements of experts. A third module is in charge of clustering the individual evaluations and reach a final decision.

The paper is organized as follows. Section 2 contains the related work tackling multi-criteria group decision making, particularly those techniques based on either fuzzy logic or MAS's. Section 3 presents the mathematical description of the AHP, fuzzy C-means and the algorithm we propose to group the individual evaluations. In Section 4 we describe the Multi-Agent System, which is then shown in Section 5. The calculations are depicted in Section 6. We finish the report showing conclusions and insights about promising future work.

2. Related work

2.1. Fuzzy approaches

It has been acknowledged that in multi-criteria decision making there is a degree of vagueness, either at the moment of making the judgements or when processing the information. On the fuzzy approach, uncertainty is measured by linguistic terms described by a given membership function. The availability of linguistic estimates may be used to evaluate alternatives by using fuzzy relations (Ekel, 2002). Similarly, fuzzy logic has been used to solve a multi-criteria problem, comparing results obtained with those of classical statistics (Kangas, Leskinen, & Kangas, 2007). An overview on how fuzzy logic has been introduced into multi-criteria decision making can be found in Wang (2000), where different proposals are set in order to employ fuzzy mathematics into the AHP. In Chan and Kumar (2007), for instance, the original scale proposed by Saaty is computed by using fuzzy numbers. Also, in Fenton and Wang (2006) the evaluator's risk and confidence attitudes are defined by a linguistic scale of triangular fuzzy numbers. Similarly, trapezoid fuzzy numbers are used to model linguistic terms on which criteria are measured, and a fuzzy distance is developed to calculate the difference between two trapezoid fuzzy numbers (Li, 2007). Fuzzy numbers are also employed to compute linguistic information provided by a group of experts (Jiang, Fan, & Ma, 2008). A fuzzy distance measure is proposed as part of a fuzzy clustering methodology for linguistic opinions when evaluations are expressed vaguely (Chakraborty & Chakraborty, 2007). A fuzzy AHP system has been developed to select machine tools by evaluating criteria with fuzzy numbers (Ayag & Ozdemir, 2006). These approaches follow earlier attempts to model the vagueness of the evaluators' judgement (Cuong, 1999). Although such proposals are highly valuable, we intend to establish patterns that come up naturally when individual evaluations form clusters, and by establishing to what degree of membership they belong to one cluster. Fuzzy C-means is suitable for our goals because it considers that any given value (crisp evaluation, in our approach) might reside on two clusters at the same time. So, to establish to what cluster such value belongs to, the higher membership degree is considered. We consider that it is entirely possible to measure the proximity of the evaluators' judgement via their membership degree to a group of similar measures within a well-defined cluster. An interesting upgrade of our MAS is to allow evaluators to express linguistic opinions rather than crisp values.

2.2. The Multi-Agent approach

As it has been stated before in this paper, in real-world situations data are acquired asynchronously, at geographically dispersed sites, and processed by decision making algorithms. On this line of research (Lee, Ghosh, & Nerode, 2000) developed a Mathematical Framework that permits the description of centralized decision making algorithms and facilitates the synthesis of distributed decision making. Even though the concept of Agency is not explicitly used in the sense of FIPA-based MAS, entities capable of establishing communication are developed. However, such communication is described in the form of signals, and intelligent behavior is not granted to such entities. Instead, utility functions are considered to synthesize the final decision. This obviously contrasts with our approach, since we are using the theory of speech acts to model conversations, and we synthesize a soft-computing technique to promote approximate reasoning. On the other hand, the usage of argumentation-based MAS has been proposed as an approach to multiple criteria group decision making. An overview of such approach is presented by Matsatsinis and Tzoannopoulos (2008). However, emphasis is placed on the argumentation and negotiation mechanisms, rather than on the agent's intelligent capabilities to reach a solution. A cooperative knowledge-based system has been designed to support decision makers who are not in the same place at the same time, enabled by cooperation processes (Soubie & Zaraté, 2005). A decision support system, enabled under web services, has been developed in order to promote distributed decision making (Yuen & Lau, 2008). Recent proposals support our claim that fuzzy clustering and MAS lead to high quality decisions (Yu, Wang, & Keung Lai, 2008). In their model, agents are given fuzzifying capabilities so that crisp evaluations are transformed into fuzzy opinions. Such fuzzified opinions are then compared and aggregated into a single group opinion. However, such approach differs from ours because they do not present a distributed solution based on the AHP, and we do not use linguistic labels to make the evaluation.

3. Methods' presentation

3.1. The analytical hierarchy process

It consists of three major stages. First, an evaluator judges the relative importance of evaluation criteria on a pair-wise basis. This leads to a pairwise comparison matrix (**PCM**), possessing the following structure:

$$PCM = \begin{vmatrix} 1 & c_{12} & \dots & c_{1p} \\ c_{21} & 1 & \dots & c_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ c_{p1} & c_{p2} & \dots & 1 \end{vmatrix}, \tag{1}$$

where c_{ij} is a numeric value that shows the relative importance of criterion c_i to criterion c_j. This first stage completes with the calculation of the eigenvector of the **PCM**.

$$eigenCriteria = \begin{vmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{vmatrix}, \tag{2}$$

Eigenvector **eigenCriteria** defines the actual priority obtained by each criterion.

On a second stage, the evaluator decides to what extent one alternative over another complies with a given criteria.

$$PCM^{criterion}_{alternative} = \begin{vmatrix} 1 & a_{12} & \dots & a_{1m} \\ a_{21} & 1 & \dots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & 1 \end{vmatrix}, \tag{3}$$

where a_{ij} is a numeric evaluation that reflects to what extent alternative a_i complies with criterion c_k when compared to alternative a_j. The eigenvector of matrix (3) is computed.

$$eigenAC_k = \begin{vmatrix} eac_{1k} \\ eac_{2k} \\ \vdots \\ eac_{mk} \end{vmatrix}, \tag{4}$$

In $eigenAC_k$,eac_{jk} represents how alternative j ranks when it is evaluated against criterion k. The second step is repeated as many times as criteria exist, terminating when all the resultant eigenvectors are arranged orderly in matrix **EIGENAC**.

The third and final step of the AHP consists of multiplying matrix **EIGENAC** times eigenvector **eigenCriteria** calculated in step one.

$$EIGENAC \cdot eigenCriteria \tag{5}$$

The result is vector **W**:

$$W = \begin{vmatrix} w_1 \\ w_2 \\ \vdots \\ w_m \end{vmatrix}, \tag{6}$$

where w_l represents the final and definite ranking obtained by each alternative. The alternative with the highest score gets the highest rank.

3.2. Fuzzy C means clustering algorithm

Data clustering is concerned with the partitioning of a data set into several groups such that the similarity within a group is larger than that among groups. A cluster centroid is a way to tell where the heart of each cluster is located. In fuzzy C-means (FCM), each data point belongs to a cluster to a degree of membership (Jang, Sun, & Mizutani, 1997). FCM employs fuzzy partitioning such that a given data point can belong to several groups with the degree of belongingness specified by membership grades between 0 and 1.

Let us define a set of n vectors, x_i,i = 1,. . .,n are to be partitioned into c fuzzy groups G_i,i = 1,. . .,c, and find a cluster center on each group such that a cost function of dissimilarity measure is minimized. Imposing normalization stipulates that the summation of degrees of belongingness for a data set always be equal to unity:

$$\sum_{i=1}^{c} \mu_{ij} = 1, \quad \forall\, j = 1,\ldots,n. \tag{7}$$

The cost function (or objective function) measures a fuzzy distance between a vector x_k in group j and the corresponding cluster center c_i, can be defined by:

$$J(U, c_1, c_2, \ldots, c_c) = \sum_{i=1}^{c}\sum_{j=1}^{n} (\mu_{ij})^m d_{ij}^2, \tag{8}$$

where μ_{ij} is between 0 and 1, c_i es the cluster center of fuzzy group i,$d_{ij} = \|c_i - x_j\|$is the Euclidean distance between ith clusters center and jth data point; and $m > 1$, is called a weighted exponent, which is judiciously chosen. Observe matrix U being defined by an $c \times n$ membership matrix, where the element $\mu_{ij} \in [0,1]$ is defined by a membership function for the jth data point x_j belonging to group i, as:

$$\mu_{ij} = \begin{cases} 1 & \textit{if } \|x_j - c_i\|^2 \leqslant \|x_j - c_k\|^2, \text{ for each } k \neq i, \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

The necessary conditions for Eq. (8) to reach a minimum can be found by forming a new objective function *bar J* as follows:

$$\begin{aligned} \bar{J}(U, c_1, c_2, \ldots, c_c, \lambda_1, \ldots, \lambda_n) &= J(U, c_1, c_2, \ldots, c_c) \\ &\quad + \sum_{j=1}^{n} \lambda_j \left(\sum_{i=1}^{c} \mu_{ij} - 1 \right) \\ &= \sum_{i=1}^{c}\sum_{j=1}^{n} \mu_{ij}^m d_{ij}^2 \\ &\quad + \sum_{j=1}^{n} \lambda_j \left(\sum_{i=1}^{c} \mu_{ij} - 1 \right), \end{aligned} \tag{10}$$

where λ_j,j = 1 to n, are the Lagrange multipliers for the n constraints in Eq. (7). By differentiating $\bar{J}(U, c_1, c_2, \ldots, c_c, \lambda_1, \ldots, \lambda_n)$ with respect to all its input arguments, the necessary conditions for Eq. (8) to reach its minimum are

$$c_i = \frac{\sum_{j=1}^{n} \mu_{ij}^m x_{ij}}{\sum_{j=1}^{n} \mu_{ij}^m}, \tag{11}$$

and

$$\mu_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{d_{ij}}{dkj} \right)^{\frac{2}{m-1}}}, \tag{12}$$

Algorithm 1. Fuzzy C means. Given the data set Z, choose the number of cluster $1 < c < N$, the weighting exponent $m > 1$, a constant for a cost function minimum $\epsilon > 0$, and a constant Th which is a termination tolerance threshold. Initialize the partition matrix U randomly, such that $\mu_{ij}(0) \in [0,1]$

Step 1. **Compute clusters prototypes:** Calculate c fuzzy cluster centers c_i, $i = 1,\ldots,c$ using Eq. (11).
Step 2. **Compute the cost function**: According to Eq. (8). Stop if either it below the tolerance ϵ or its improvement over previous iteration is below the threshold Th.
Step 3. **Compute a new**: U using Eq. (12). Go to **Step 2**.
End of the FC-Means algorithm.

3.3. Clustering evaluators data

We describe the usage of fuzzy C-means and the AHP to process evaluators data and reach a conclusion that best represents the group expertise.

Let $\xi = \{e_1, e_2, \ldots, e_n\}$ be the evaluators set. Each x: $i \in \xi$ must compare the relative importance of a finite set of criteria $C = \{c_1, c_2, \ldots, c_p\}$ and the relative importance of $A = \{o_1, o_2, \ldots, o_l\}$ options or alternatives. Criteria evaluation yields:

$$PCM^k = \begin{vmatrix} 1 & a_{12}^k & \ldots & a_{1p}^k \\ a_{21}^k & 1 & \ldots & a_{2p}^k \\ \vdots & \vdots & \vdots & \vdots \\ a_{p1}^k & a_{p2}^k & \ldots & 1 \end{vmatrix}, \tag{13}$$

where $k = 1, 2, \ldots, n$ is the kth evaluator's judgement; a_{ij}^k is the relative importance of criterion i over criterion j as determined by evaluator e_k. When all the n pairwise comparison matrices for criteria are formed, it remains to construct the global pairwise comparison matrix PCM^G.

The algorithm to construct the global pairwise comparison matrix is as follows:

1. The cardinality p of set C is computed.
2. A matrix PCM^G of dimensions $p \times p$ is formed.
3. The diagonal of matrix PCM^G is filled with 1.
4. Vector α_{ij} is formed with entries $a_{ij}^k, k = 1, 2, \ldots, n$.

5. $a_{ij}^G = FuzzyCMeans(\alpha_{ij})$.
   Method *countIncidences* is called for determining the quantity of evaluators inside each cluster. Cluster with the highest number of incidences is selected. Cluster centroid is obtained.
   If number of incidences are equal for both clusters, then method *calculateClusterDispersion* is executed. Cluster with the minor dispersion wins.
6. Repeat steps 4 and 5 $\forall(i,j) = 1,2,\ldots,p$; $\forall(PCM^k)$, $k = 1, 2,\ldots,n$

Thus,

$$PCM^G = \begin{vmatrix} 1 & a_{12}^G & \ldots & a_{1p}^G \\ a_{21}^G & 1 & \ldots & a_{2p}^G \\ \vdots & \vdots & \vdots & \vdots \\ a_{p1}^G & a_{p2}^G & \ldots & 1 \end{vmatrix}. \quad (14)$$

Eq. (14) is the resultant global pairwise comparison matrix for criteria. Entries a_{ij}^G are the centroid values of the winning clusters.

Next, we illustrate how a Multi-Agent System fully automates the processing of data. Specifically, the entire set of activities, from data gathering, processing and final calculation is performed by the distributed and intelligent system.

4. The Multi-Agent System

This section depicts the Multi-Agent System structure and dynamics. The structure of the MAS is fixed by the following agents:

- A *coordinator agent*,
- A set of *evaluator agents*,
- A *clustering agent*,
- An *decisor agent*.

The following activities are performed by the ensemble of agents:

1. *Coordinator agent* acquires problem variables i.e. evaluation criteria and alternatives, as well as the number of evaluators required. It leaves a message on the *Evaluation Blackboard* to inform each of the *evaluator agents* about the newly input problem.
2. *Evaluator agent* reads the *Evaluation Blackboard* to acquire the problem parameters, informing these data to the human evaluator. Then, it receives the absolute judgements for both, criteria and alternatives, and constructs the corresponding pairwise comparison matrices. It stores the resultant PCM matrices in the *Evaluation Blackboard*.
3. *Coordinator agent* verifies that every *evaluator agent* has completed his/her task.
4. *Coordinator agent* informs *clustering agent* upon verification of data completeness. Then, *clustering agent* formes vector α_{ij} (see step 4 of the proposed algorithm) and processes such vector with fuzzy C-means.
5. *Clustering agent* constructs the global pairwise comparison matrices for both, criteria and alternatives, by calculating the centroid value of the largest cluster. This centroid is entry a_{ij}^G.
6. *Clustering agent* informs *coordinator agent* upon completion of its assignment.
7. *Coordinator agent* requests *decisor agent* to compute the AHP. Then, it informs when the task is achieved.
8. *Coordinator agent* revises the *Evaluation Blackboard* to obtain the final result of the decision process.

The implementation of the MAS is done on the JADE platform (Bellifemine, Caire, & Greenwood, 2007). JADE is a useful tool because it allows to promote intelligent behavior to a given agent, while providing a rich set of communication capabilities based on FIPA-ACL. Both, the fuzzy C-means clustering technique and the AHP were developed on Java, and agents call such coding transparently. The MAS is a distributed architecture because each agent resides in its own processing unit, and communication is done over the TCP/IP protocol, for which JADE possesses powerful libraries.

The structure of the MAS is shown in Fig. 1 by means of a deployment diagram. The previous list of activities is formally represented in the communication diagram of Fig. 2. Those two types of diagrams are part of UML 2.0 (Bauer & Odell, 2005).

As it can be seen in Fig. 1, *coordinator agent* communicates directly with both, *clustering agent* and *decisor agent*. It is not so regarding the *evaluator agents*. In this latter case, communication is done by posting messages on the *Evaluation Blackboard*. This *Evaluation Blackboard* is represented in Fig. 2 as an *artifact*. Such blackboard is actually a database implemented on MySQL, whose structure is shown in Fig. 3.

5. Experimental results

In this section we present a case-study to validate the agent-based DSS. The case study refers at selecting a new robotic manipulator for the manufacturing department. Top management must

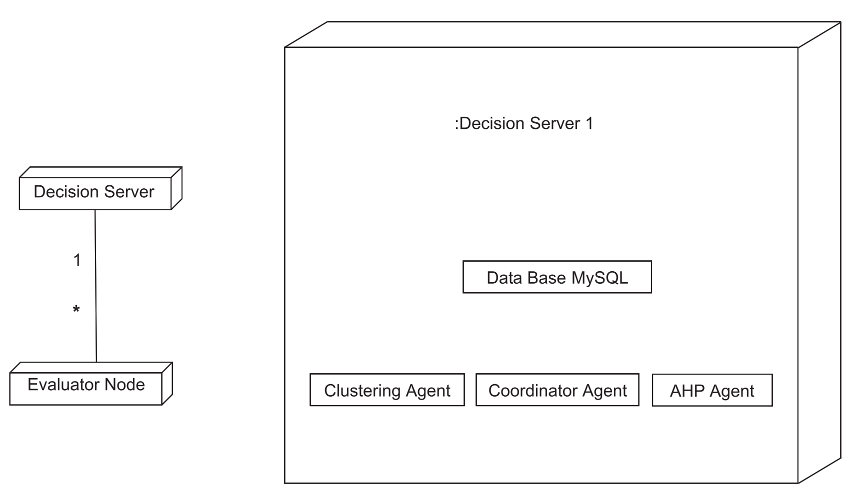

Fig. 1. Structure of the Multi-Agent System.

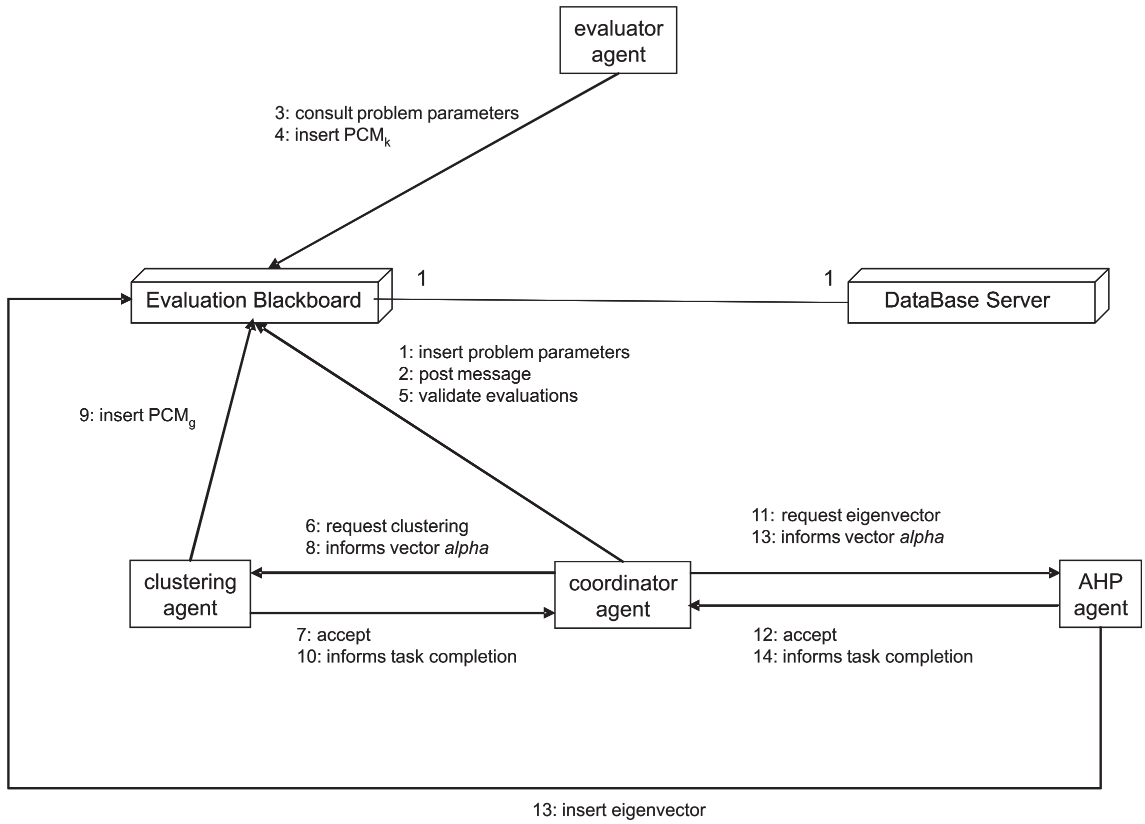

Fig. 2. Communication diagram of the Multi-Agent System.

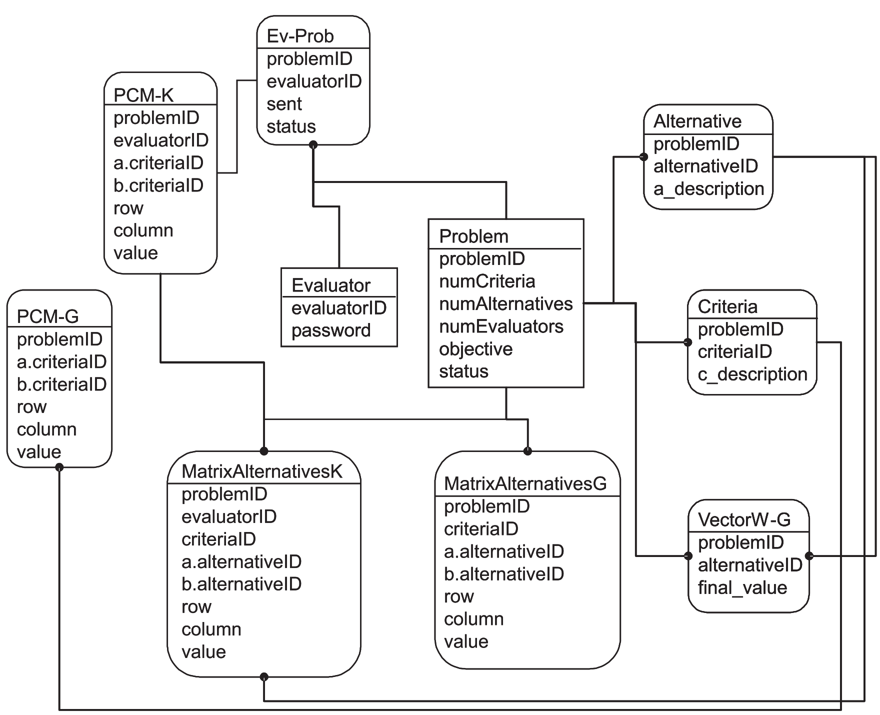

Fig. 3. IDEF1x model of the Evaluation Blackboard.

purchase the robotic manipulator that best reflects post-sale service, performance precision, impact on productivity, and overall technological capabilities. Ten individuals, each having different background and responsibilities, were selected by top management from the manufacturing, finance and sales departments to evaluate the alternatives. Data adapted from (Rao, 2007), for which we include nicknamed robots' manufacturers, is the basis of the evaluation. In the following table we present the attributes associated to the seven alternatives.

R_i	Robots' features				
	Load	Repeat. error	Tip speed	Reach	Manuf.
R_1	60	0.4	2540	990	e1
R_2	6.35	0.15	1016	1041	e2
R_3	6.8	0.1	1727.2	1676	e2
R_4	10	0.2	1000	965	e3
R_5	2.5	0.1	560	915	e4
R_6	4.5	0.08	1016	508	e3
R_7	3	0.1	1778	920	e4

- Load capacity: kilograms.
- Repeatability error: millimeters.
- Tip speed: millimeters per second.
- Reach: millimeters.

Thus, $\xi = \{e_1, e_2, \ldots, e_{10}\}$ conforms the set of evaluators; $C = \{c_1, c_2, c_3, c_4\}$ is the set of criteria where: c_1 = post-sale service, c_2 = impact on productivity, c_3 = precision, and c_4 = technological capabilities. Finally, seven different alternatives are evaluated, which are labeled $R_1, \ldots, R_7$.

Top management launches the MAS and introduces problem parameters. Firstly, top management establishes the ID associated with the problem, along with the number of criteria, alternatives and evaluators. Afterwards, He/She introduces the objective of the problem, description of criteria, and the names associated with the alternatives (Fig. 4).

These parameters are stored in Table *Problem* of the *Evaluation Blackboard*. A summary is shown in Fig. 5. Once the problem parameters are introduced, the *coordinator agent* posts a message on the *Evaluation Blackboard*, which will be read by each of the *evaluator agents*.

On its own network location, each *evaluator agent* constantly verifies whether a new problem has been introduced (this is achieved by a Ticker Behaviour). When a new problem is encountered, its parameters are displayed so that the evaluator proceeds to determine the absolute importance of every criterion. Here we would like to elaborate on this way of evaluation. According to empirical usage of the system, human evaluators complaint about the time consuming process and the inability to keep track of their own judgements when they were requested to pair-wise compare both, criteria and alternatives. They also expressed that the numbers they were facing lacked meaning at some point. Instead, all of them agreed that it is more intuitive to make an absolute judgement on a 1–10 scale (see Fig. 6).

Evaluator agent constructs the actual PCM for criteria (and for alternatives as well) based on the absolute judgements entered by the human evaluator. This is further explained in Section 6. Every evaluator also judges to what extent each alternative complies to the evaluation criteria (see Figs. 7 and 8).

The PCM matrix for criteria and the PCM's matrices for alternatives are stored in the *Evaluation Blackboard*. *Evaluator agent* posts a

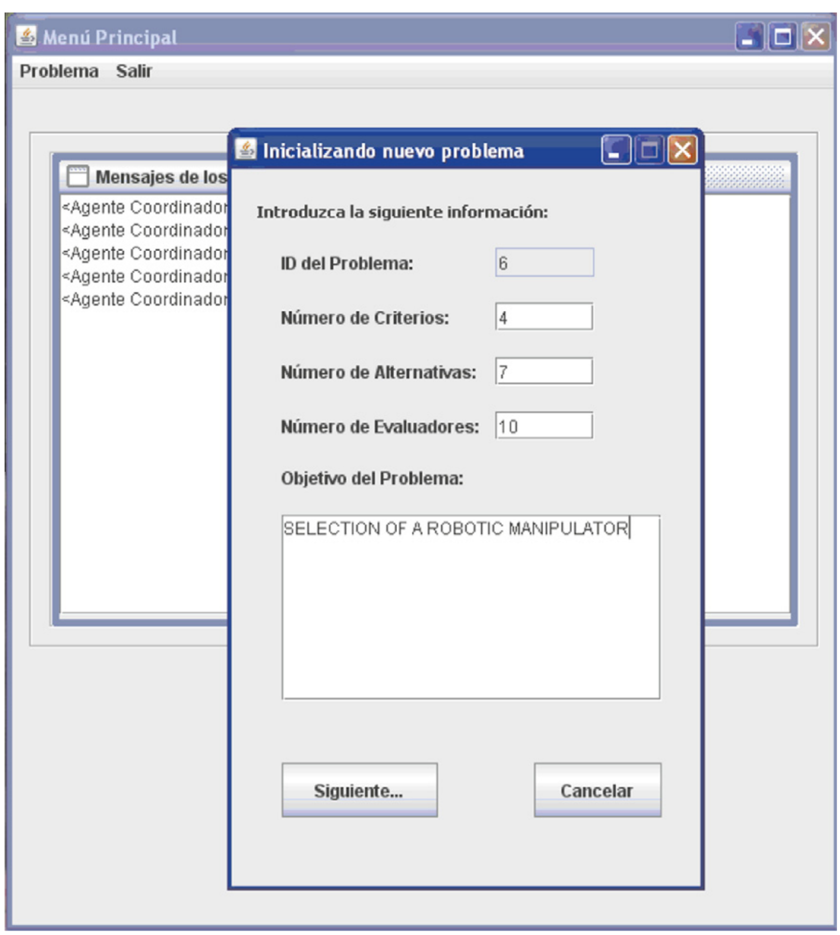

Fig. 4. Coordinator agent. Entering problem parameters.

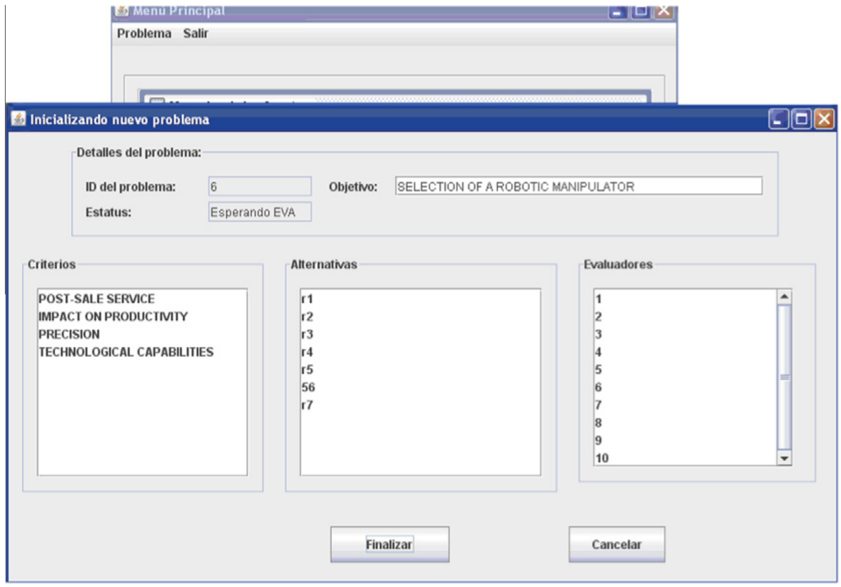

Fig. 5. Coordinator agent. Summary of problem parameters.

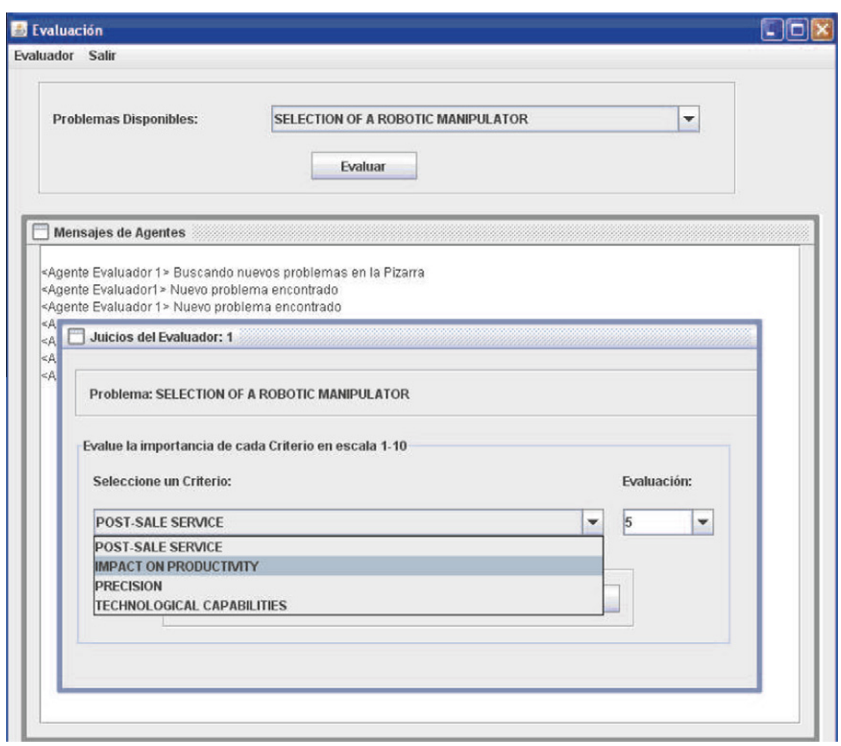

Fig. 6. Evaluator agent. Evaluation of criteria.

message informing that the problem has been evaluated successfully by a given expert.

Upon completion of the entire set of evaluations, *coordinator agent* informs *clustering agent* that it must initiate the calculation of the clusters. *Clustering agent* acknowledges receipt and proceeds to calculate the largest cluster and its centroid value associated to every vector α_{ij}. *Clustering agent* stores the Global PCM's for criteria and alternatives in the *Evaluation Blackboard*.

Shortly after the clustering process is complete, *decisor agent* executes the AHP as if it were a single evaluator, obtaining the final decision according to the group evaluation. Final results for this particular case are displayed in Figs. 9 and 10.

6. Data processing in detail

This section presents the clustering process in detail. Even though the AHP requires the creation of pairwise comparison matrices for both, criteria (Eq. (1)) and alternatives (Eq. (3)), we present only the evaluation regarding criteria. Thus, the clustering

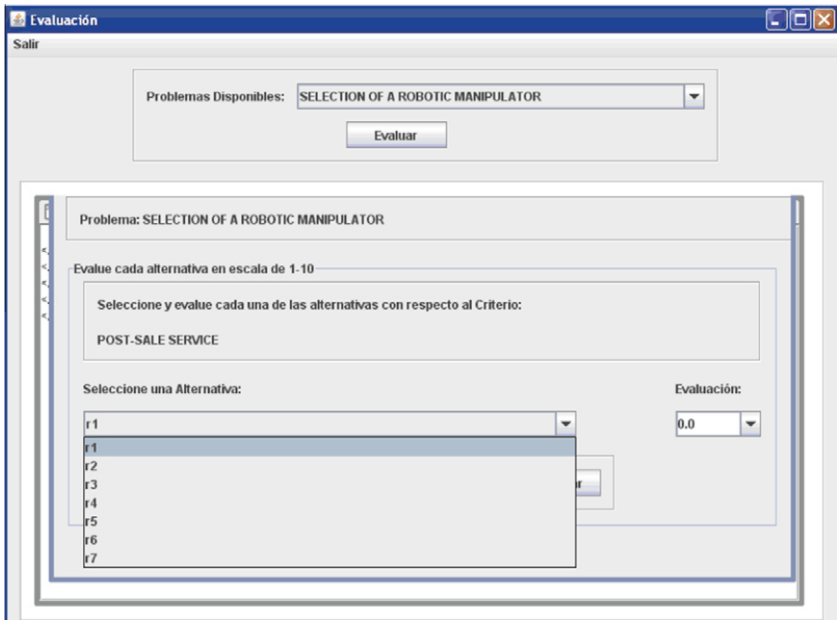

Fig. 7. Alternatives evaluation for post-sale service.

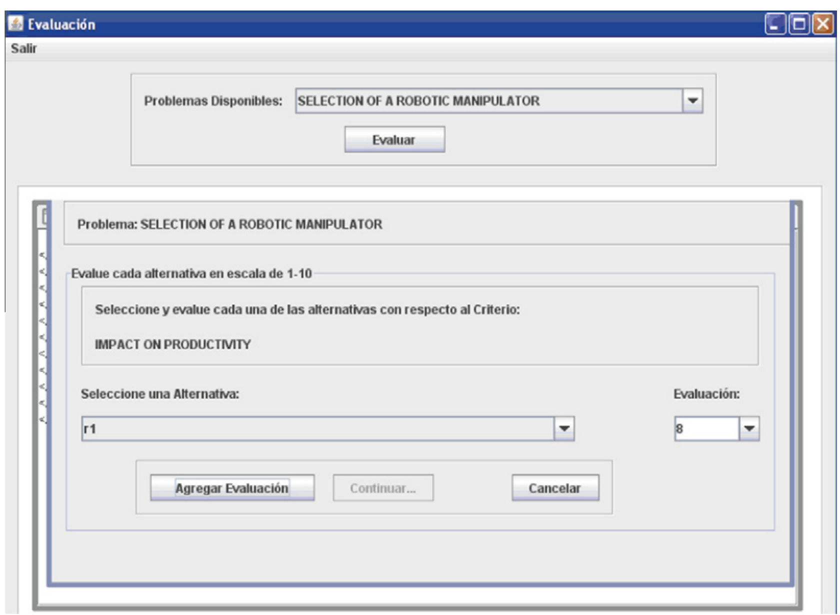

Fig. 8. Alternatives evaluation for impact on productivity.

process yields a global pairwise comparison matrix (PCM^G) and its corresponding eigenvector. A similar process is conducted to form the global matrices for alternatives.

The individual evaluations established by the group of ten experts are summarized in the following table.

c_i	e_k									
	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
c_1	5	6	9	6	4	8	8	5	3	8
c_2	9	8	9	5	5	9	10	10	10	8
c_3	7	6	7	7	6	5	6	6	5	4
c_4	7	9	7	9	7	6	8	10	9	2

To form the corresponding PCM it is proceed as follows. Evaluator 1 (e_1), for instance, assigned five points to the post-sale service criterion (c_1), nine points to impact on productivity (c_2), seven points to precision (c_3), and seven points to technical capabilities (c_4). Thus, entry a_{12} for PCM^1 measures how important c_1 is with respect to c_2. This results in a ratio 5/9 = 0.555. This process is repeated in order to construct the pairwise comparison matrix that corresponds to $e_1(PCM^1)$. This task is conducted by *evaluator agent*.

Evaluator agent receives data from every k evaluator, then it computes the relative importance of criteria, and it builds the PCM^k ($k = 1,\ldots,10$). The resulting entries for PCM^k are:

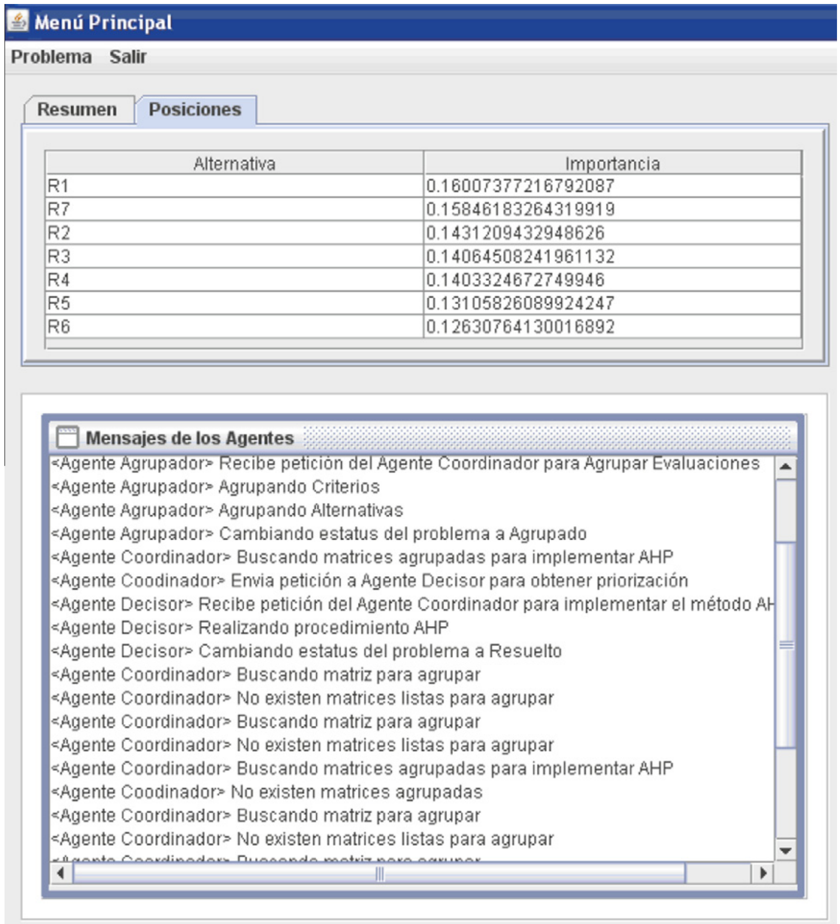

Fig. 9. The final ranking of alternatives.

e_i	a_{ij}					
	a_{12}	a_{13}	a_{14}	a_{23}	a_{24}	a_{34}
e_1	0.555	0.714	0.714	1.285	1.285	1.0
e_2	0.75	1.0	0.666	1.333	0.888	0.666
e_3	1.0	1.285	1.285	1.285	1.285	1.0
e_4	1.2	0.857	0.666	0.714	0.555	0.777
e_5	0.8	0.666	0.571	0.833	0.714	0.857
e_6	0.888	1.6	1.333	1.8	1.5	0.8333
e_7	0.8	1.333	1.0	1.666	1.25	0.75
e_8	0.5	0.8333	0.5	1.666	1.0	0.6
e_9	0.3	0.6	0.333	2.0	1.111	0.555
e_{10}	1.0	2.0	4.0	2.0	4.0	2.0

To determine the global pairwise comparison matrix it is necessary to calculate the centroid of the largest cluster, for every entry $a^k_{ij} \in PCM^k$. According to the proposed algorithm (sub Section 3.3), such entries form vector α_{ij}, which is the actual input of the fuzzy C-means. To illustrate this step, we take entries $a^k_{12} \in PCM^k$:

$$\alpha_{12} = \{0.555, 0.75, 1.0, 1.2, 0.8, 0.888, 0.8, 0.5, 0.3, 1.0\}.$$

Fuzzy C-means on vector α_{12} yields that the largest cluster has a centroid value $centroid_{12} = 0.6467$, with $N = 6$ incidences. Elements of this winning cluster are:

$$winningcluster_{\alpha_{12}} = \{0.555, 0.75, 0.8, 0.8, 0.5, 0.3\}.$$

This means that six out of ten experts provided evaluations that are centered around 0.6467. This is the actual value for entry $a^G_{12} \in PCM^G$, for criteria. In this approach the group majority is respected. Four out of ten evaluations are left out.

Clustering agent repeats this process for $a^k_{ij} \in PCM^k$ entries. The following table shows the resultant centroid values of the winning clusters, the number of experts that form such cluster and, when necessary, the lesser dispersion when both clusters possess equal number of elements:

	a_{ij}					
	a^G_{12}	a^G_{13}	a^G_{14}	a^G_{23}	a^G_{24}	a^G_{34}
Centroid	0.6467	1.039	0.805	1.4586	1.145	0.7303
Number of experts	6	6	7	5	8	7
Dispersion	na	na	na	0.33	na	na

Once every vector α_{ij} is processed through the fuzzy C-means algorithm, *clustering agent* has completed the creation of PCM^G. Eq. (15) shows the relative importance of criteria after all the individual pairwise comparison matrices are clustered.

$$PCM^G = \begin{vmatrix} 1 & 0.6467 & 1.039 & 0.805 \\ 1.3414 & 1 & 1.4586 & 1.145 \\ 0.7618 & 0.6557 & 1 & 0.7303 \\ 1.7004 & 0.9152 & 1.3984 & 1 \end{vmatrix}. \quad (15)$$

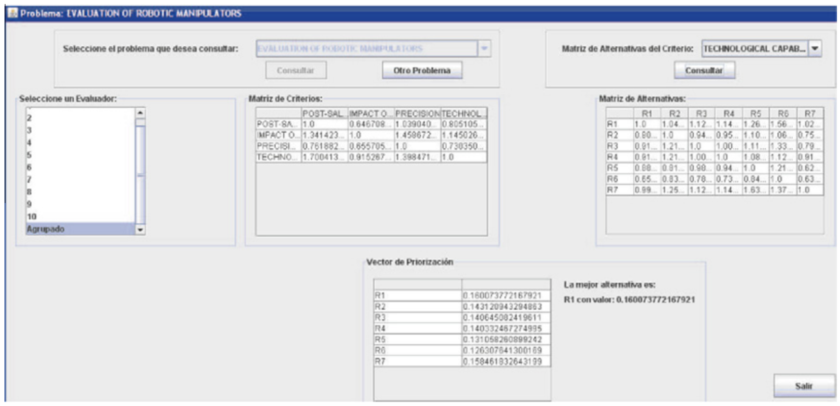

Fig. 10. summary of final results.

Upon completion of its task, *clustering agent* informs *coordinator agent*, whom immediatly *coordinator agent* instructs *decisor agent* to execute the AHP (Section 3) as if the group were a single evaluator. For the case study, Eq. (16) is the eigenvector associated to the global pairwise comparison matrix of criteria (Eq. (15)):

$$W^G = \begin{vmatrix} 0.2105 \\ 0.3004 \\ 0.1908 \\ 0.2982 \end{vmatrix}. \tag{16}$$

Hence, the majority of the ten evaluators agreed that c_2 (Impact on productivity) is the criterion that must prevail when selecting the robotic manipulator. Figs. 9 and 10 present the result of the group multi-criteria decision process.

7. Conclusions and future work

Decision making is a complex process, particularly when it is carried out by a multidisciplinary group of experts. On the other hand, companies are exploring state of the art decision support systems in order to promote distributed and asynchronous decision making.

The agent-based DSS we present in this article proved useful to achieve the former goals. Our agent-based DSS is a platform where top management can fix problem parameters, spread them through the company, and solve any group multi-criteria decision problem. At this regard, fuzzy C-means, used as a pre-processing step, not only helps determining the aggregated PCM^G's for both criteria and alternatives, but it is also useful to establish which evaluators actually agree on the resultant figures. This is an important piece of knowledge in order to provide feedback, and to elucidate afterwards reasons for discordant evaluations. Altogether, the distributed and intelligent system that we propose is useful for organizations that desire to improve multi-criteria group decision making.

After reviewing the related literature, we have encountered opportunities for upgrading the DSS. We can improve the evaluation process by using linguistic assessments rather than asking for absolute evaluations. On the theoretical side, we envision the calculation of fuzzy distances by means of fuzzy operators.

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